



VIII Congreso Nacional y
I Congreso Internacional
de Riego, Drenaje y Biosistemas
COMEI - UAAAN 2023 | Saltillo, Coahuila
4 al 6 octubre 2023



CONFERENCISTA MAGISTRAL

Matemáticas y el Agua.

Dr. Fernando Brambila Paz



Fecha de presentación: 06 de octubre de 2023



AGRICULTURA
SECRETARÍA DE AGRICULTURA Y DESARROLLO RURAL



Sonhos
universidad personalizada



Historia reciente.

Sandoval Vallarta. MIT, CONACYT, ICTP.
la Revolución.

Salio de México en

En la segunda guerra mundial trae a México: G. Birkhoff Harvard.
N. Wiener MIT.
S. Lefschetz. Princeton.

Fundan en Saltillo la SMM, 30 de junio 1943. Participa el ex rector
Remigio Valdés Gámez.

S. Lefschetz se queda en México y es profesor de Arturo Fregoso en la
F.C. UNAM y lo manda hacer su doctorado a Illinois.

Dr. Arturo Fregozo regresa a México y enseña Matemáticas y sus
aplicaciones agrícolas.

Dr. Arturo Fregozo enseña en UNAM, UAM, UACH, Colegio de
Posgraduados.

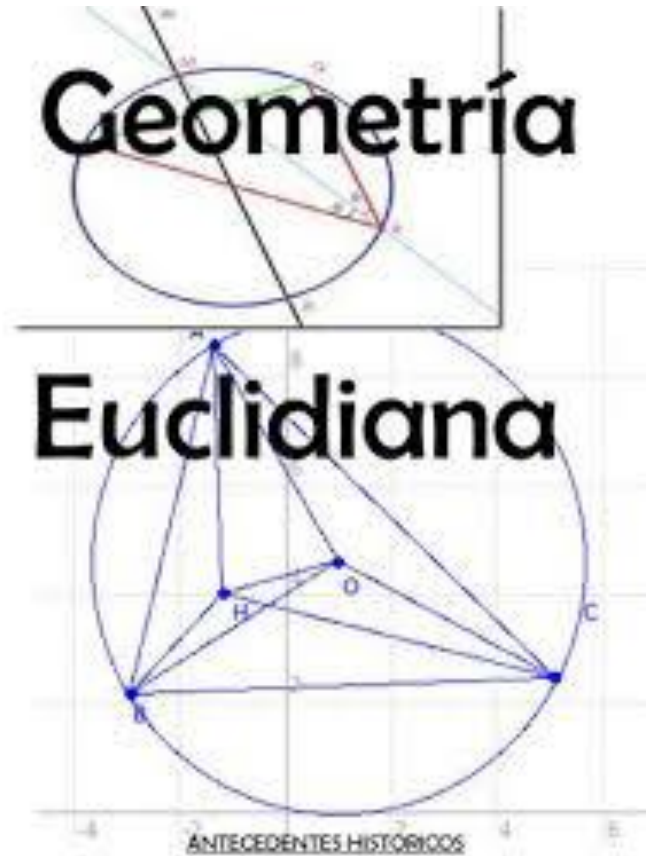


Modelo matemático de Derivadas Fraccionarias.

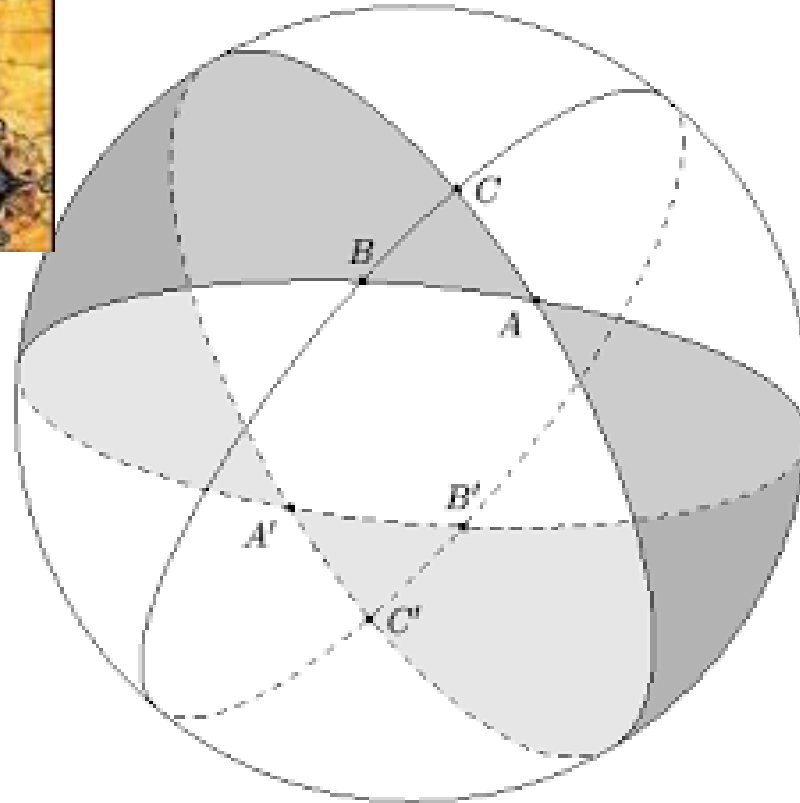
Newton Rhapsion Fraccionario.

Fenómenos representados con Ecuaciones Parciales fraccionarias. Y sus soluciones.

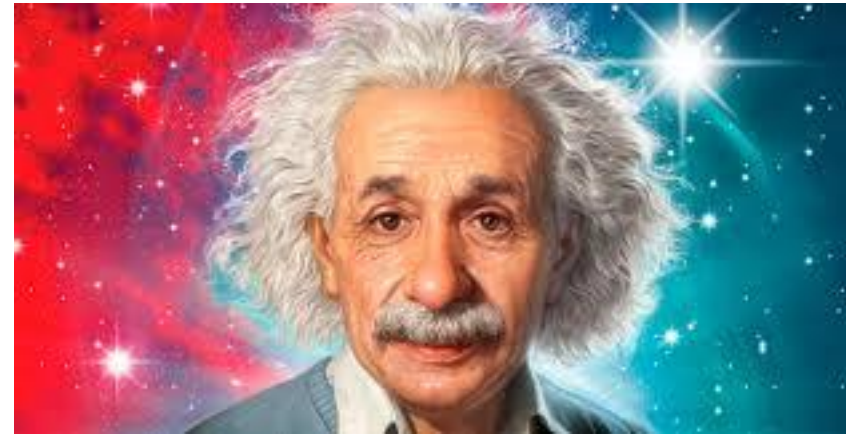
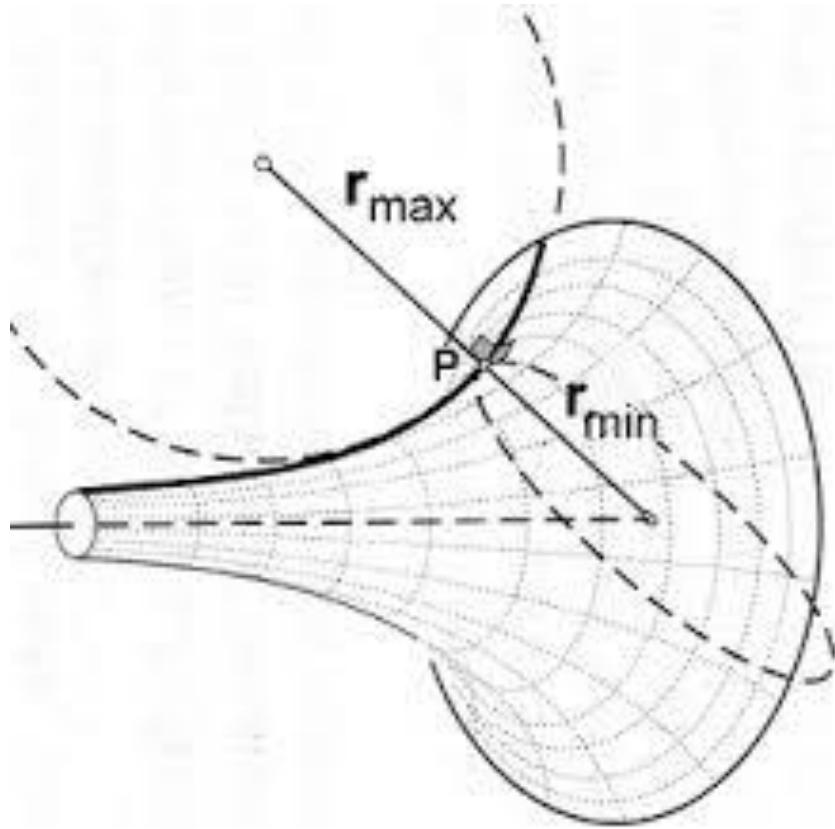
Geometría Griega



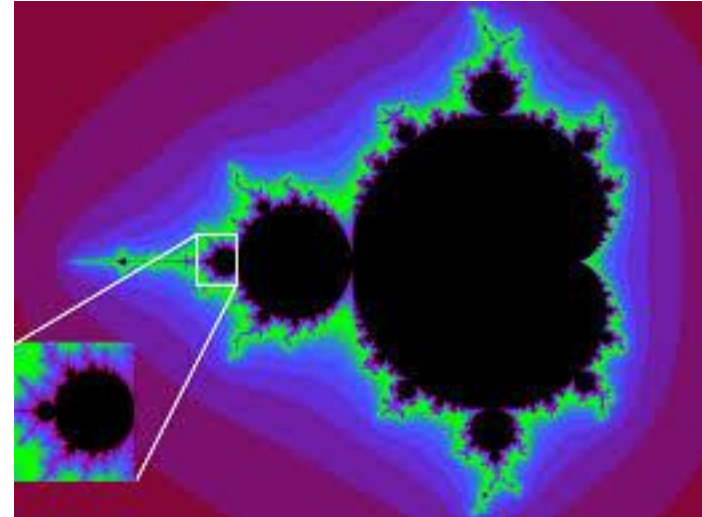
Geometría esférica



Geometría Hiperbólica



Geometría fractal



Historia de geometría y su cálculo.

- Geometría Euclides.
- Geometría esférica.
- Geometría hiperbólica.
- Geometría Fractal.
- Cálculo diferencial.
- Ecuaciones parciales diferenciales.
- Cálculo Fraccionario
- Ecuaciones parciales fraccionarias.

Contents

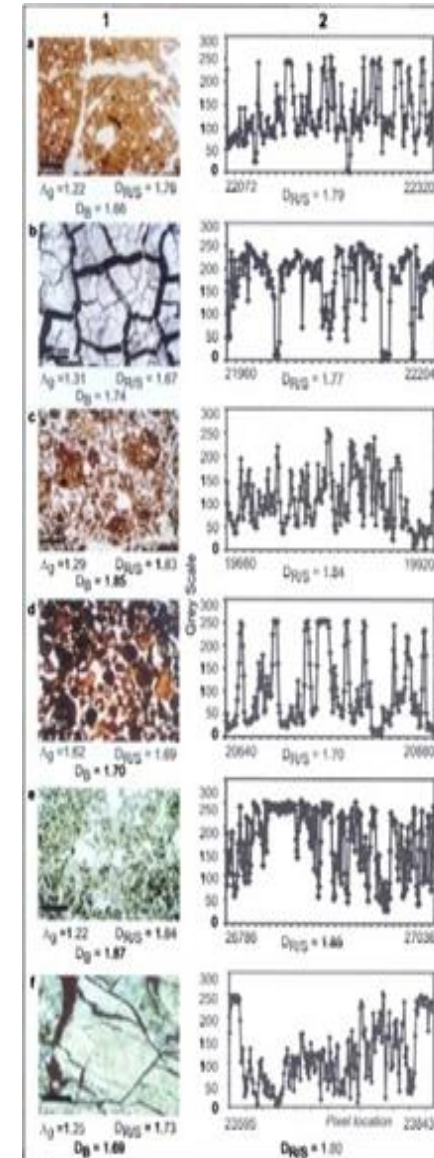
- 1.- Porosity = Fractal Dimension.
- 2.- Fractal Geometry versus Fractional Calculus.
- 3.- Fractional Partial Differential Eq.

1.- Porosity = Fractal Dimension. K. Oleschko

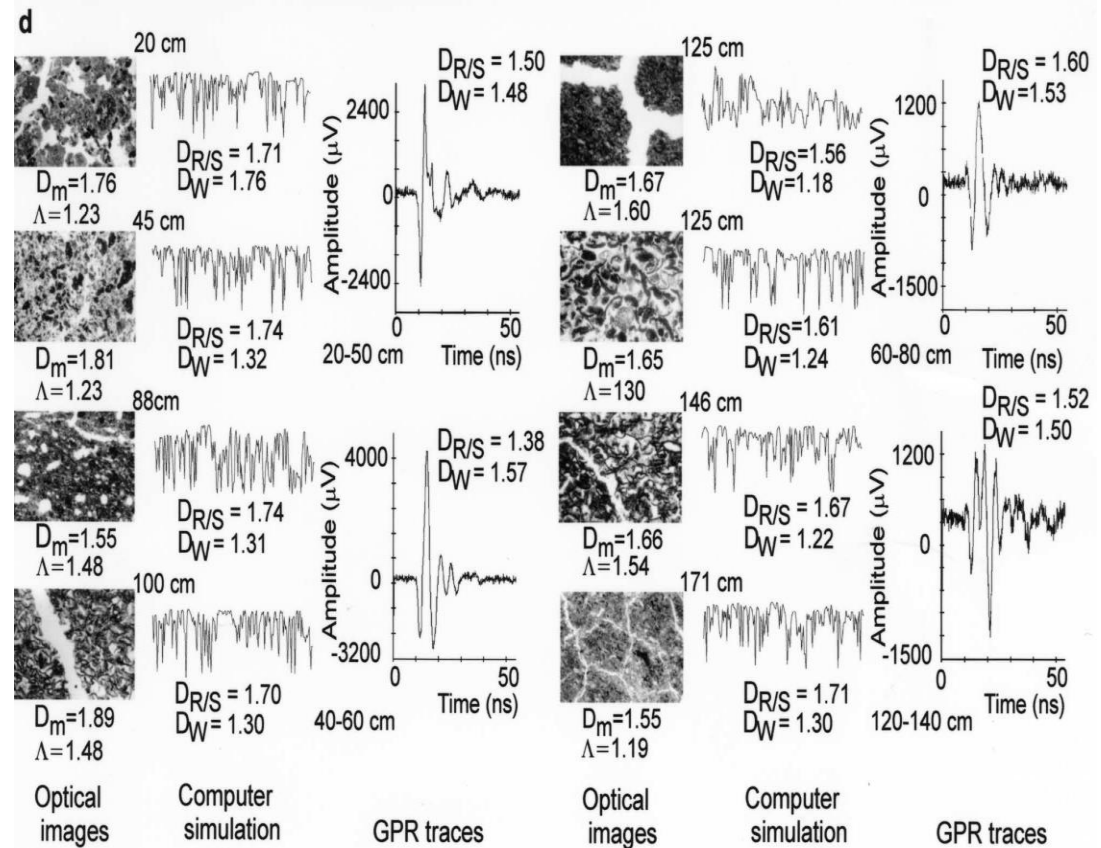
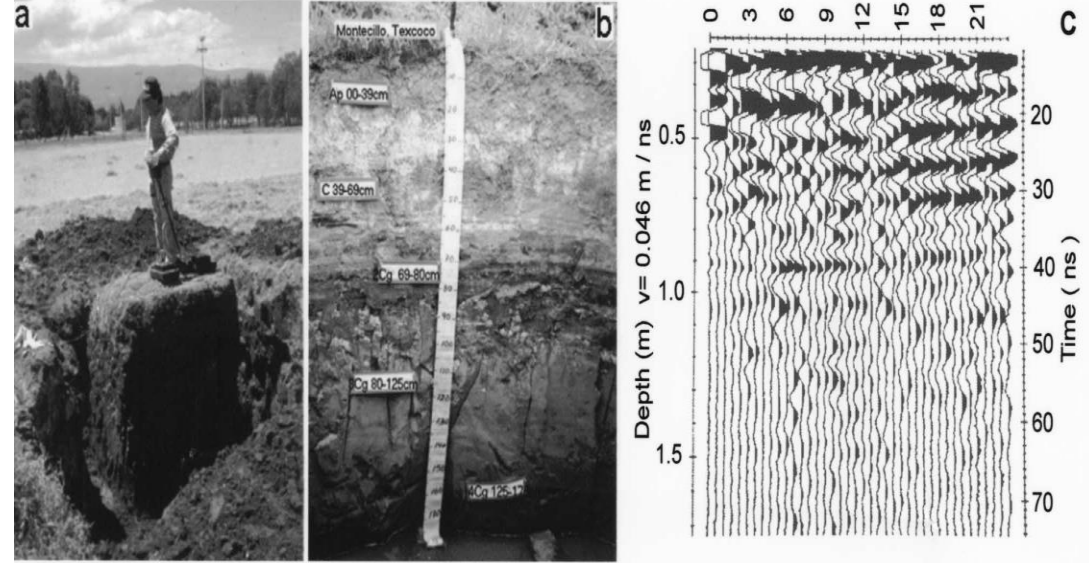
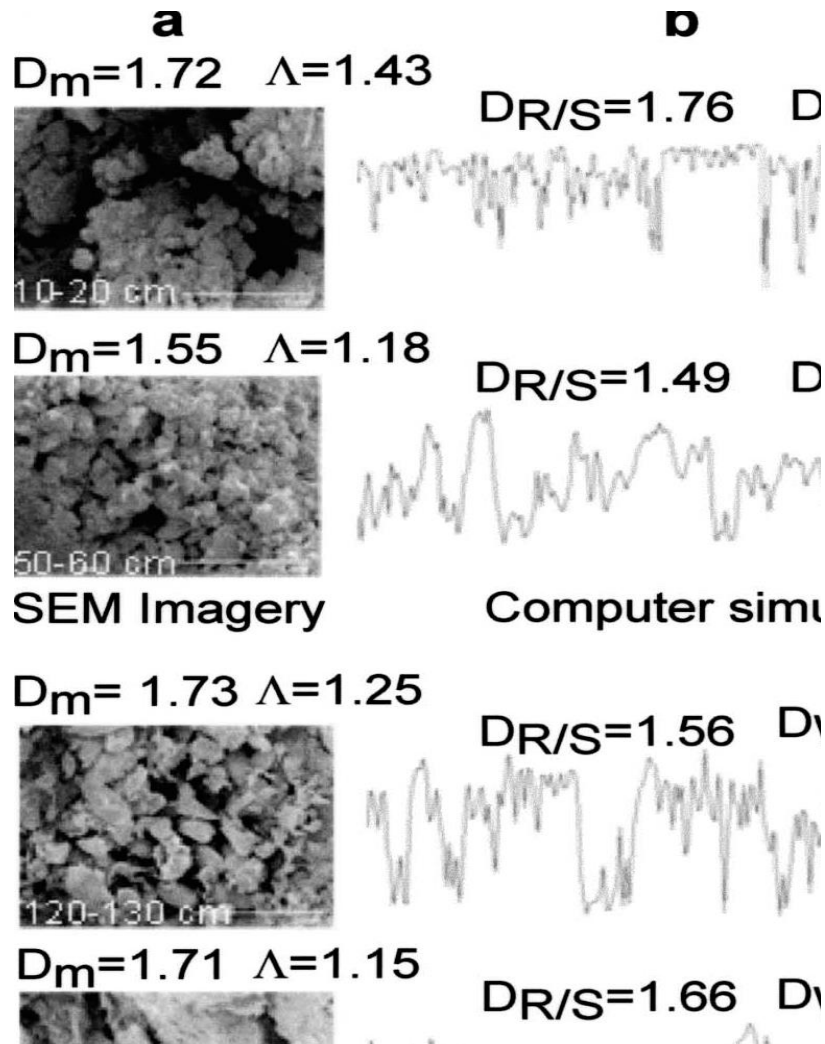
- **Soil (Tomography)**

- Use ground-penetrating radar (GPR) image (georadargrama)
- Fractal dimension of upper amplitude
 - correlated with fractal dimension of the porous media.

Scattering of
electromagnetic waves

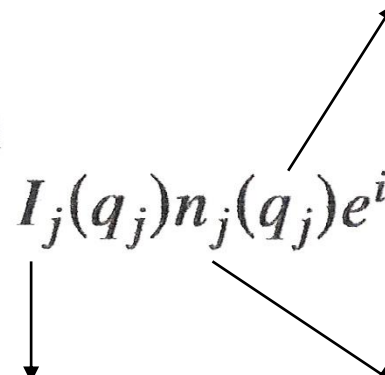


Electronic microscope



III Mathematical Model

Scattering of electromagnetic waves

$$A(t) \propto \sum_{j=1}^{N \gg 1} I_j(q_j) n_j(q_j) e^{iq_j ct},$$


$$\lim_{j \rightarrow \infty} \frac{\ln A_j}{\ln |q_j|} = -H < 0,$$

Hausdorff dimension

(fractal-dimension) $A(t)$

$$D = 2 - H$$

Article

Physical Review Letters

Volume 8, Number 18

October 2002

***Fractal Scattering of
Microwaves from Soils***


K. Oleschko,¹ F. Brambila,² G. Korvin,³ et. al.

¹Instituto de Geología, UNAM

²Facultad de Ciencias, Departamento de Matemáticas, UNAM

³King Fahd, University of Petroleum and Minerals

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News

October 16, 2002

Radar Can Probe Soil's Structure

Determining whether or not a swath of soil will lend itself to agricultural use usually requires extensive testing, which may disrupt the soil and can be time-consuming. Now a report published in the October 28 issue of *Physical Review Letters* outlines a quick, noninvasive way of estimating soil's suitability for planting. Researchers have used microwave radar to determine the physical and mechanical properties of a section of earth without disturbing it.



Image: LOURDES FLORES DELGADILLO

Klaudia Oleschko of the National Autonomous University of Mexico and her colleagues determined that although soil structure is extremely heterogeneous, on the scale of one centimeter to one meter it is also a fractal. That is, patterns that can be described with relatively simple equations are repeated over varying size scales. The scientists exploited this property to identify the volume of empty space—a good indicator of density and water content—within a sample of dirt. Microwaves reflected by a soil sample, the team found, had the same fractal dimensions as the sample itself. When the researchers used microwaves to scan a cube of earth with six very different layers (*see image*), the results obtained from the radar technique agreed well with direct measurements taken from each of the sections. The authors conclude that ground-penetrating radar is "a prominent tool for nondestructive soil studies." —Sarah Graham

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Backscattered Ground Penetrating Radar traces gives:

- Pore topology
- Dielectric Permittivity
- Water content
- Bulk density
- Mechanical Resistance
- Roughness

Problem

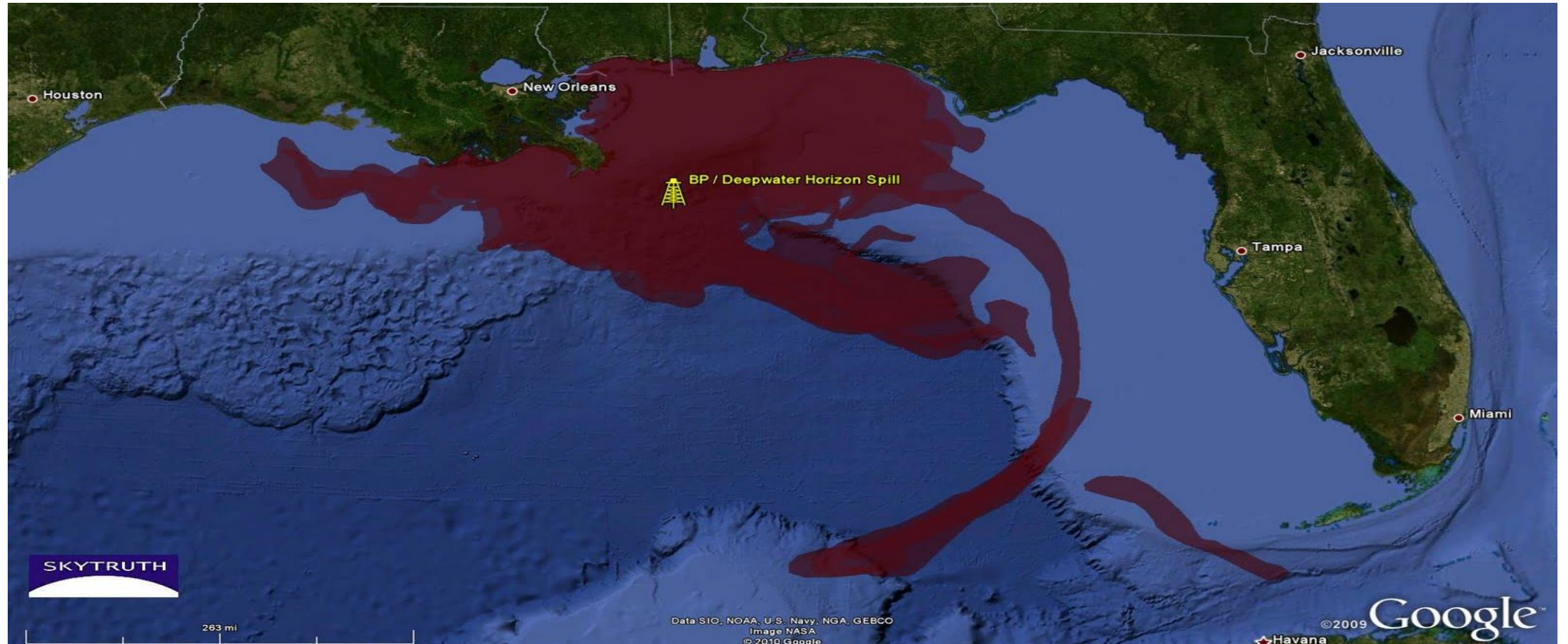
C. Fuentes. K Oleschko.

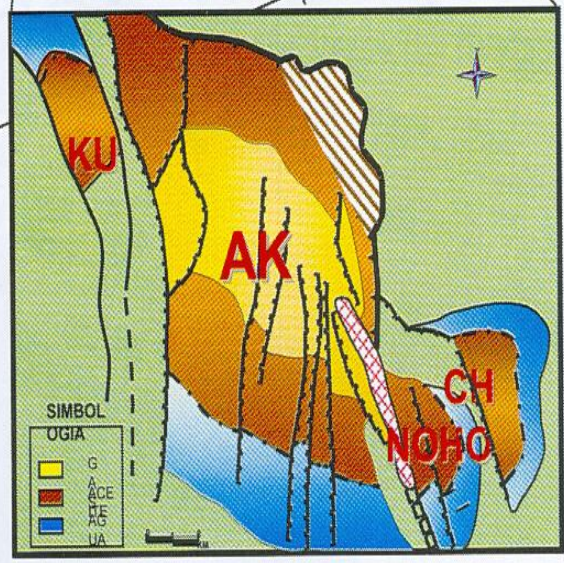
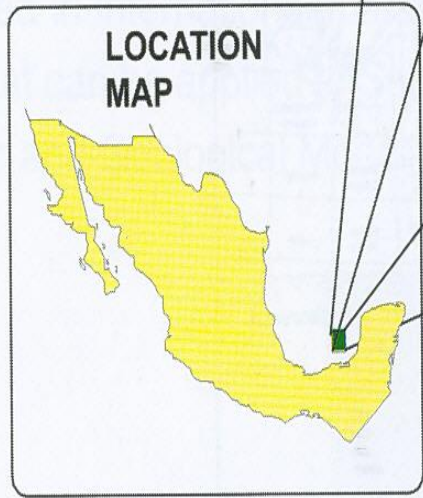
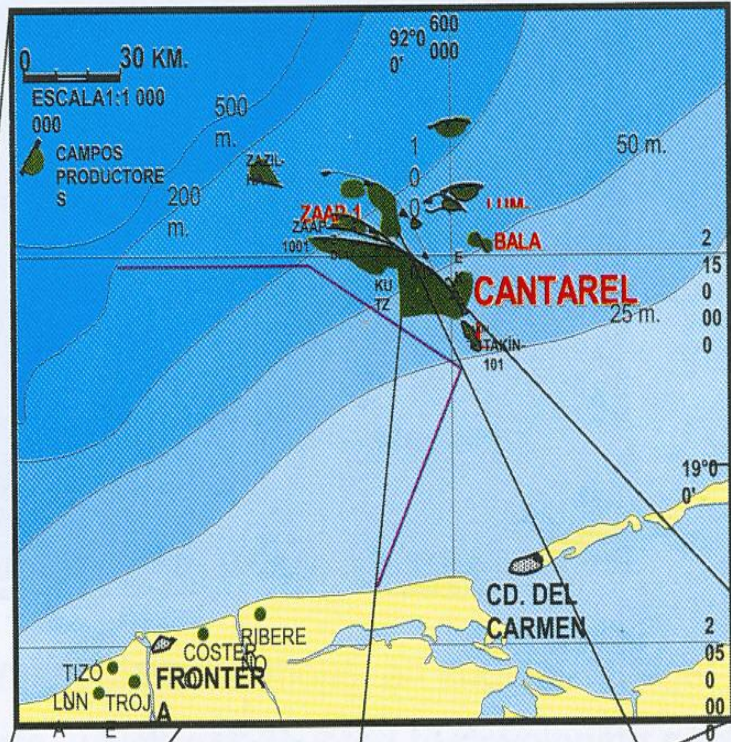
2010: gigantic oil spill in the Gulf of Mexico – caused by an accident on a British Petroleum (BP) deep-sea exploration rig.

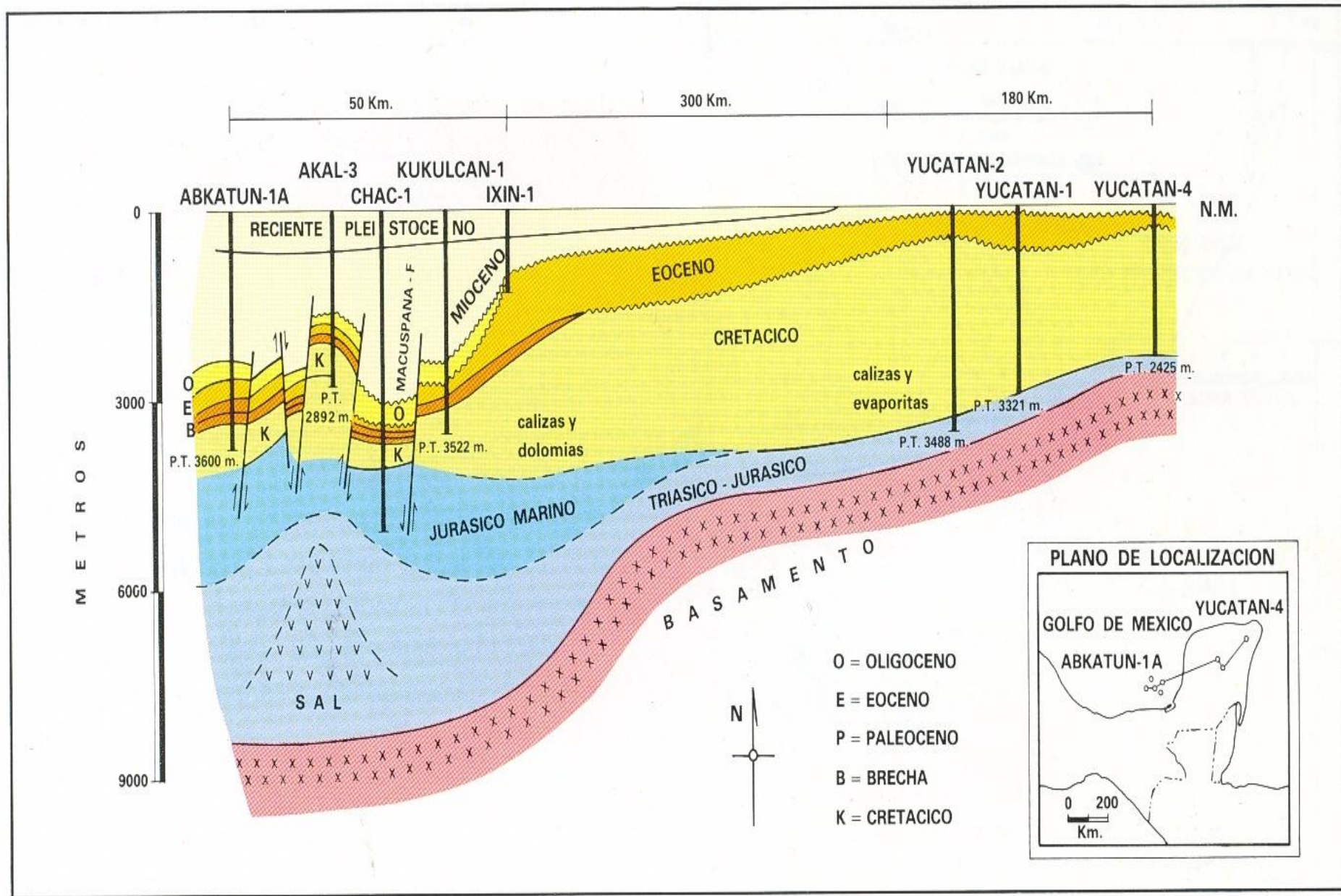
- Multi-million losses to businesses
- The worst environmental disaster in America.



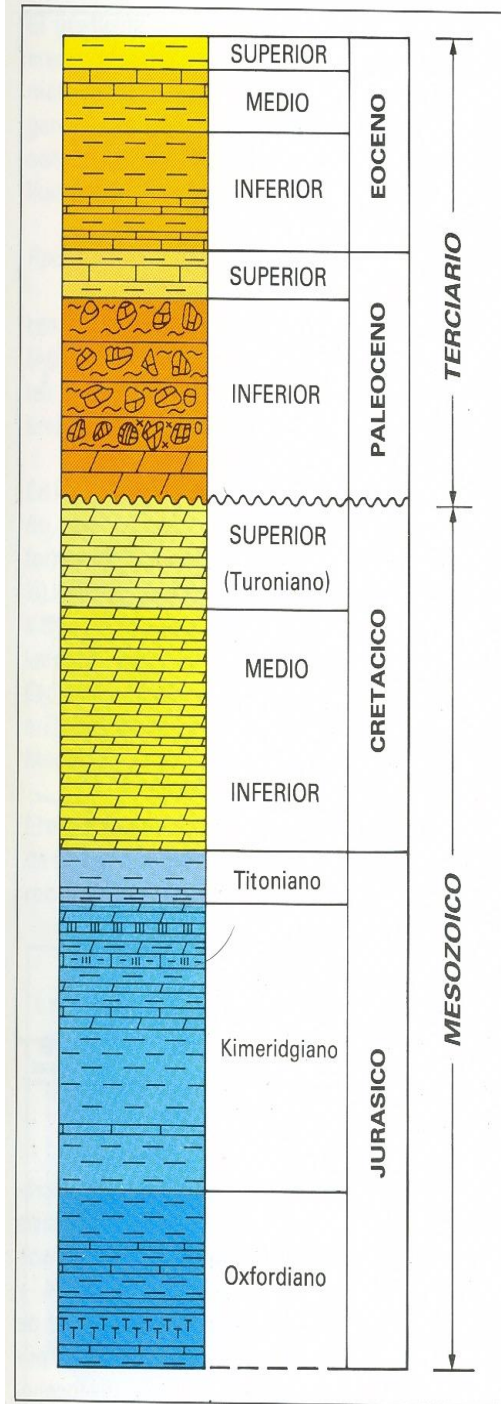
Problem





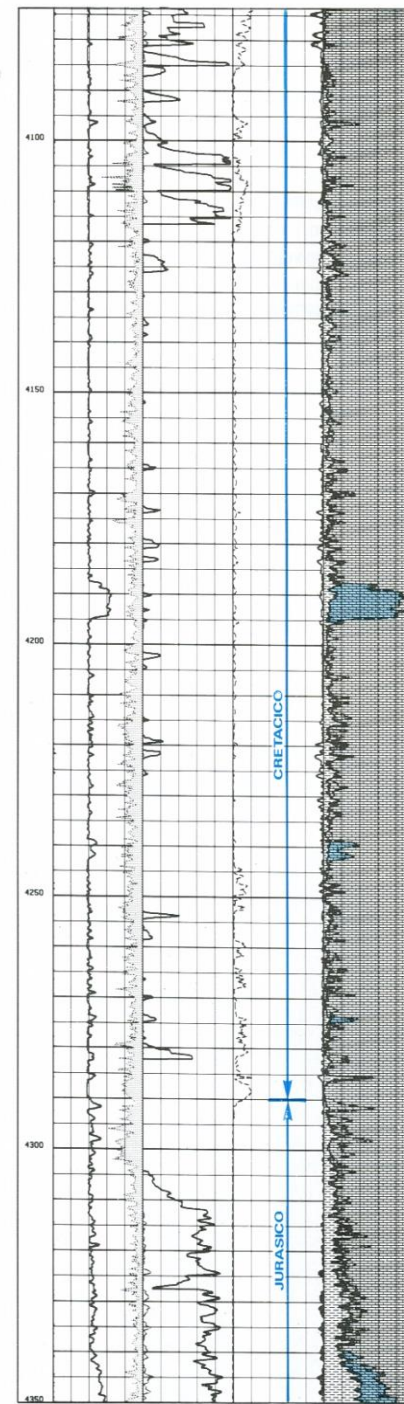
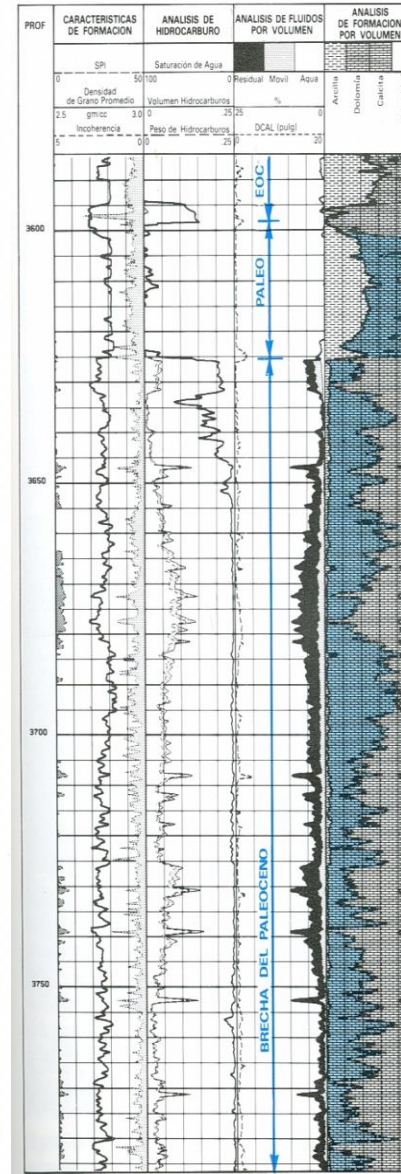


Stratum Porosity



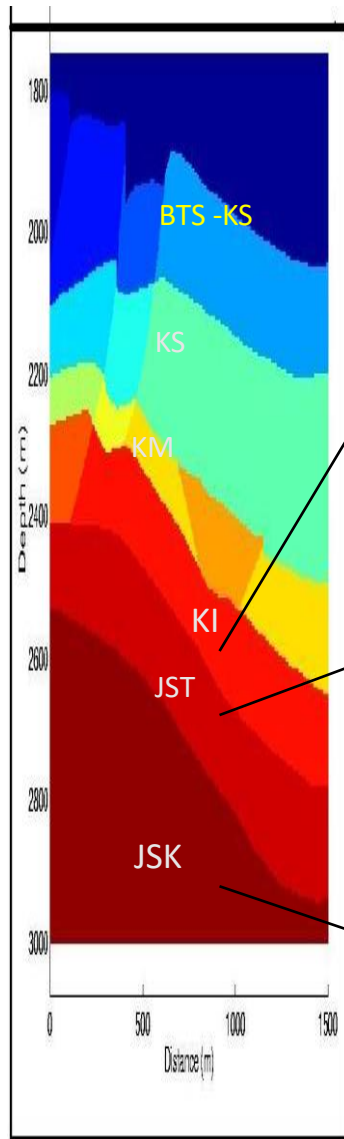
SONDA DE CAMPECHE

La columna típica de la Sonda de Campeche se ha dividido para fines prácticos en las siguientes unidades estratigráficas: brechas del Eoceno, brecha del Paleoceno, Cretácico, Jurásico

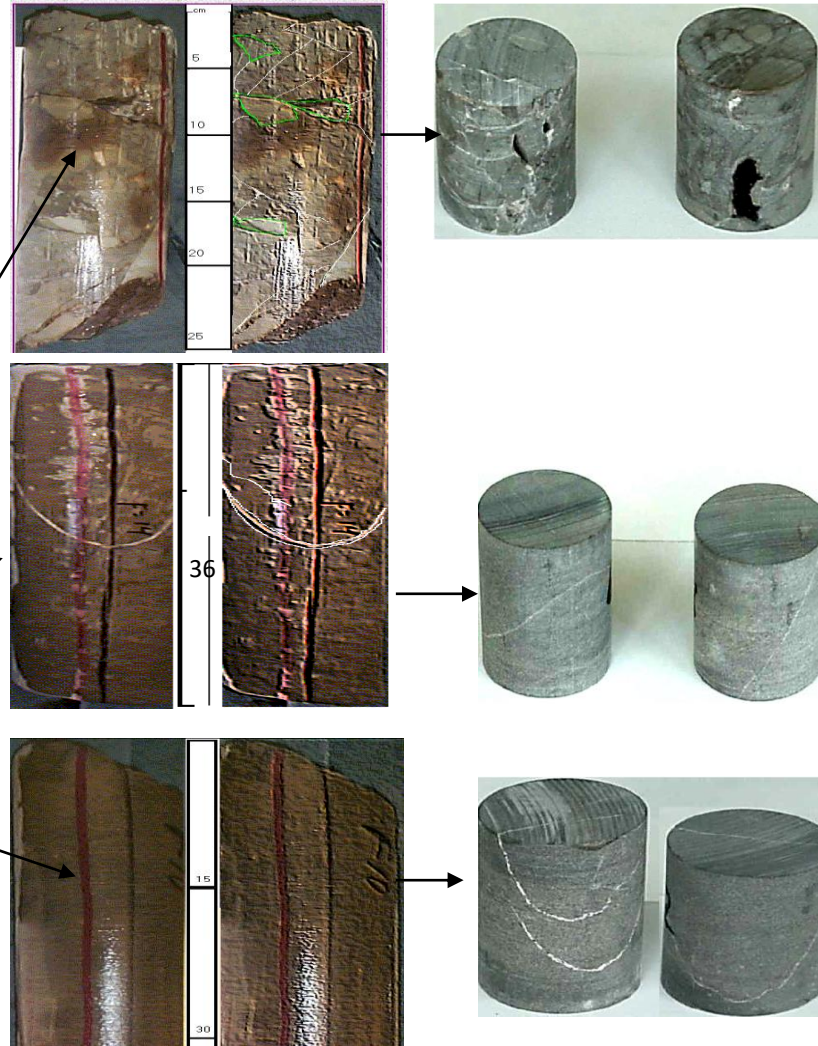


Análisis multiescalar

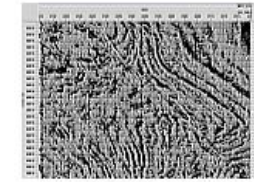
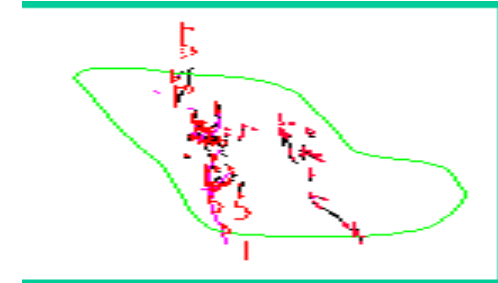
Yacimiento



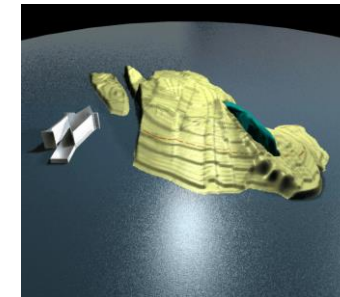
núcleos



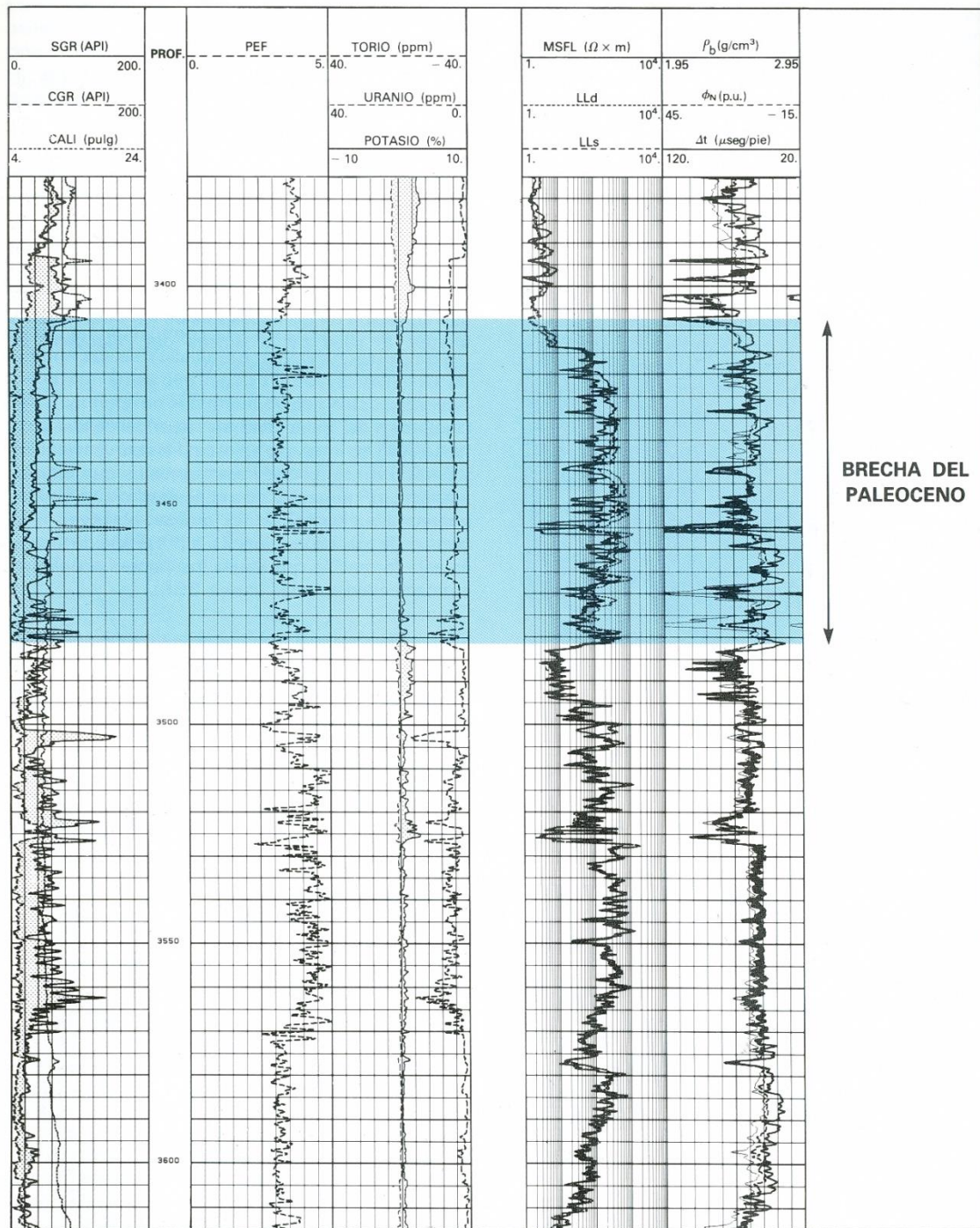
sísmica



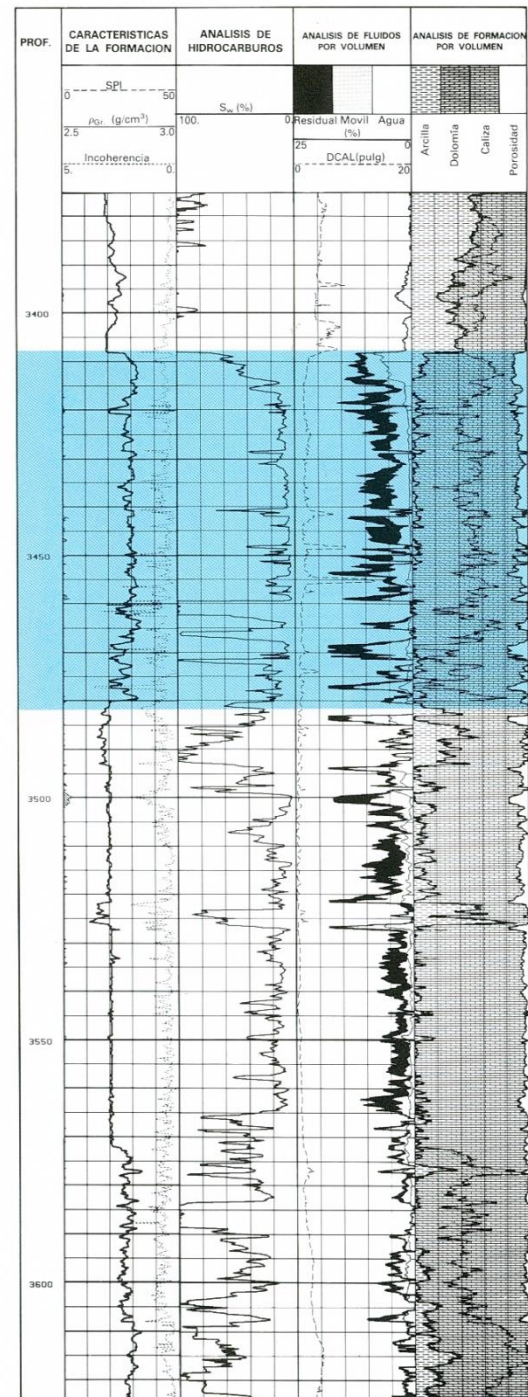
1 s.



1.8 s.



BRECHA DEL PALEOCENO



Conclusions

- The minimum fractal dimension is at the cretaceous period
- The trace at the cretaceous age represents least rugosity
- We found that the fractal dimension is **correlated** with resistivity, radioactivity, density and porosity
- The rugosity is less when the soil is more fractal
- It takes **less time** to do the analysis

2.- Fractional derivatives.

300 years of History

1695 *L'Hopital* asks *Leibniz*: "What if the derivate is fractional?"

Leibniz replied: This is an apparent paradox from which, one day, useful consequences will be drawn."

Newton: Physical interpretation?

Geometrical interpretation?

300 years of History

1730 *Euler*: “...With the help of interpolation, one may be able to define fractional derivatives”.

1772 *Lagrange*: Fractional differential operators.

1812 *Laplace*: Fractional integral and fractional derivatives.

300 years of History

1820 *Fourier*: Fractional derivatives $\sin(x)$,
 $\cos(x)$, e^x .

Fourier transform- Weak Derivatives.

1847 *Riemann-Liouville*: Integral-fractional operators.

Abel, Caputo, Grunwald,...

Physical interpretation 1967-2018.
interpretation 1992-2018.

Geometrical

Euler 1730

Euler (1730) introduce la función Gamma. Euler sabía que necesitaba generalizar (o interpolar como él decía) el producto $1 \cdot 2 \cdots n = n!$ para valores de n no enteros, y el propuso una integral

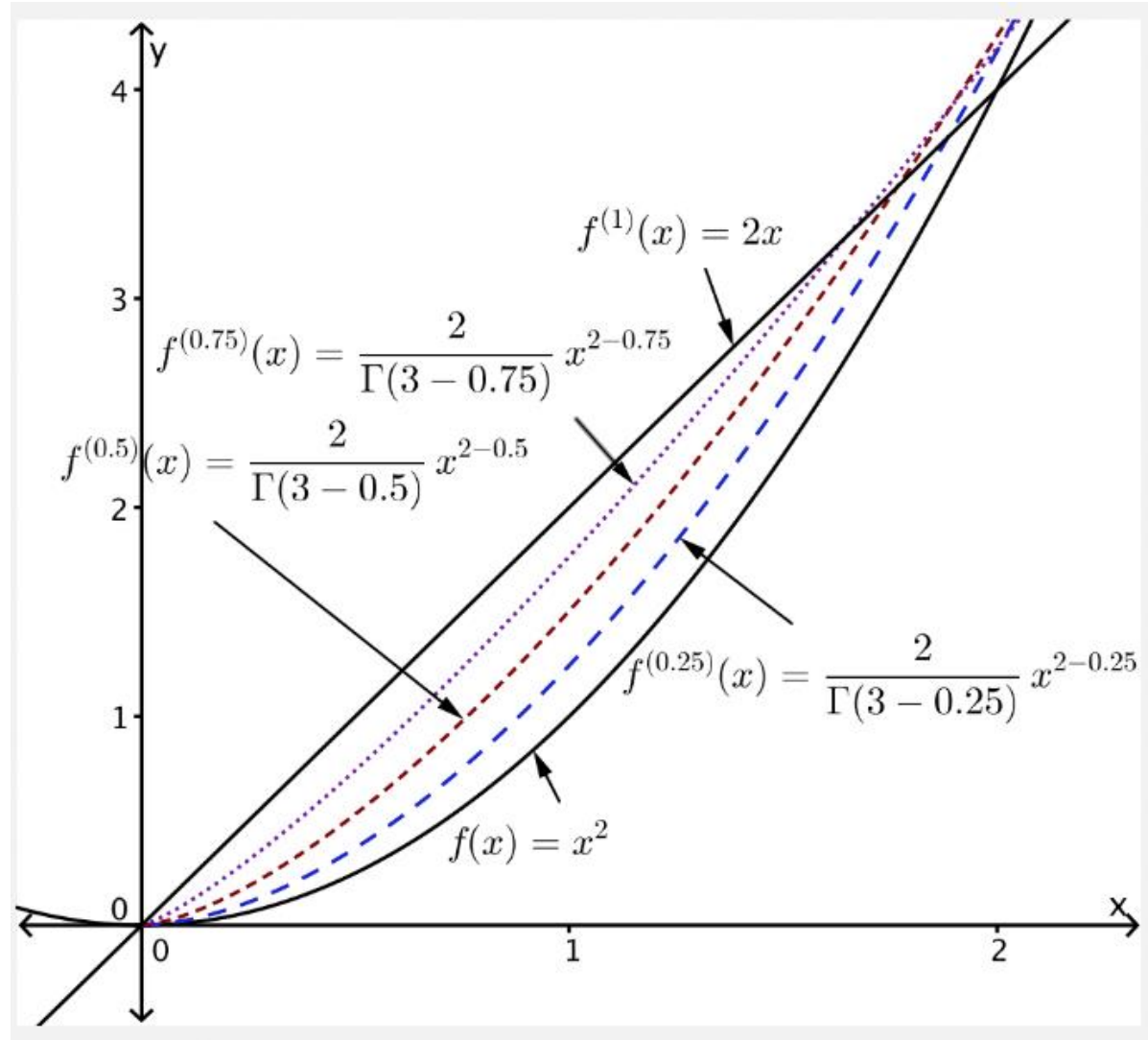
$$\prod_{k=1}^n k = \int_0^1 (-\log x)^n dx, \quad (1)$$

y la utilizó para resolver parcialmente la paradoja de Leibniz, el dio la derivada fraccional básica (la notación moderna con $\Gamma(n + 1) = n!$)

$$\frac{d^\alpha x^\beta}{dx^\alpha} = \frac{\Gamma(\beta + 1)}{\Gamma(\beta - \alpha + 1)} x^{\beta - \alpha}, \quad (2)$$

válida para α y β no enteros

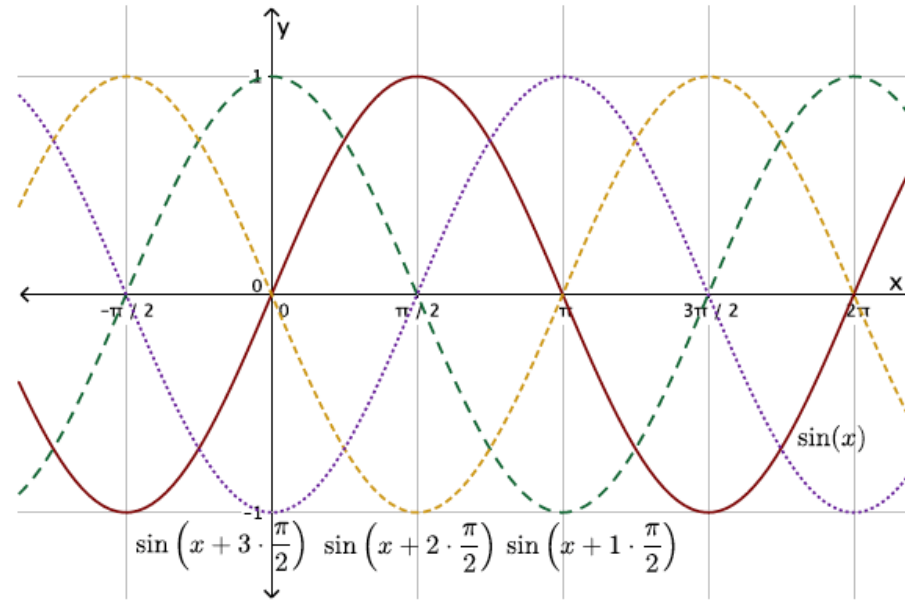
Fractional Derivative 1730.
Physical interpretation?
Geometrical interpretation?



1820 Fourier.

Derivada fraccionaria de $\sin(x)$, $\cos(x)$, exponencial.

$$\begin{aligned} f(x) &= \sin x \\ \frac{\partial f}{\partial x} &= \sin\left(x + 1 \cdot \frac{\pi}{2}\right) \\ \frac{\partial^2 f}{\partial x^2} &= \sin\left(x + 2 \cdot \frac{\pi}{2}\right) \\ \frac{\partial^3 f}{\partial x^3} &= \sin\left(x + 3 \cdot \frac{\pi}{2}\right) \\ &\vdots \\ \frac{\partial^n f}{\partial x^n} &= \sin\left(x + n \cdot \frac{\pi}{2}\right) \end{aligned}$$



Si α es un número **no** natural ¿Por qué no $\frac{\partial^\alpha f}{\partial x^\alpha} = \sin\left(x + \alpha \cdot \frac{\pi}{2}\right)$?

$$\begin{aligned}
 f(x) &= \text{sen } x \\
 \int f(x) dx &= \text{sen} \left(x - 1 \cdot \frac{\pi}{2} \right) \\
 \int \left(\int f(x) dx \right) dx &= \text{sen} \left(x - 2 \cdot \frac{\pi}{2} \right) \\
 \int \left(\int \left(\int f(x) dx \right) dx \right) dx &= \text{sen} \left(x - 3 \cdot \frac{\pi}{2} \right) \\
 &\vdots \\
 \int \cdots \int f(x) dx^n &= \text{sen} \left(x - n \cdot \frac{\pi}{2} \right)
 \end{aligned}$$

Si α es un número **no** natural ¿Por qué no

$$\int \cdots \int f(x) dx^\alpha = \text{sen} \left(x - \alpha \cdot \frac{\pi}{2} \right)?$$

Fourier Transform

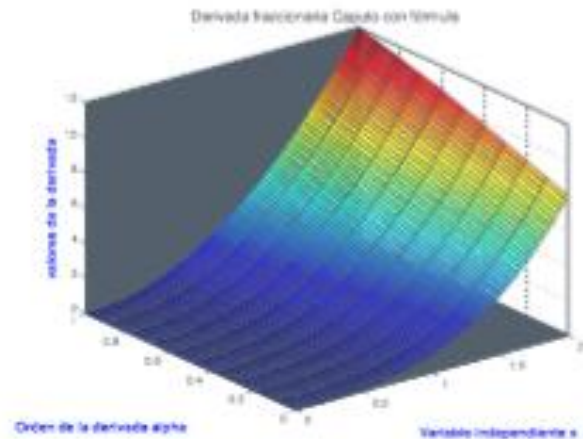
We can define weak derivative of any order using integration by parts.

!920-1930 is used to find weak derivative of Dirac delta function. (Quantum Mechanics)

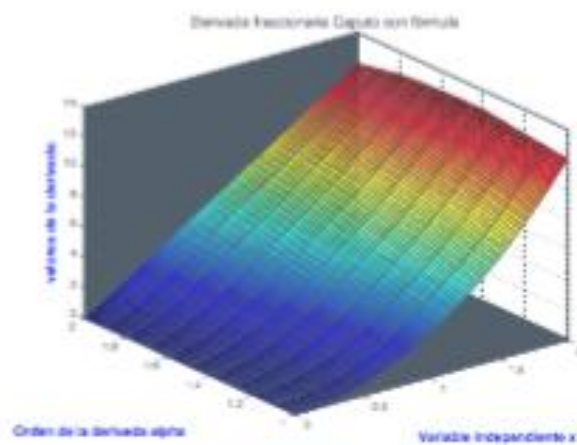
A lot of function will have weak derivative.

The Cantor function C does not have a weak derivative.

Figura: Derivadas fraccionarias de x^3 , donde el orden va de 0 a 2

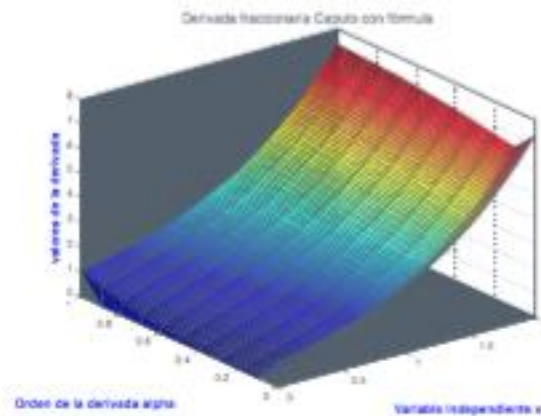


(a) orden de 0 a 1

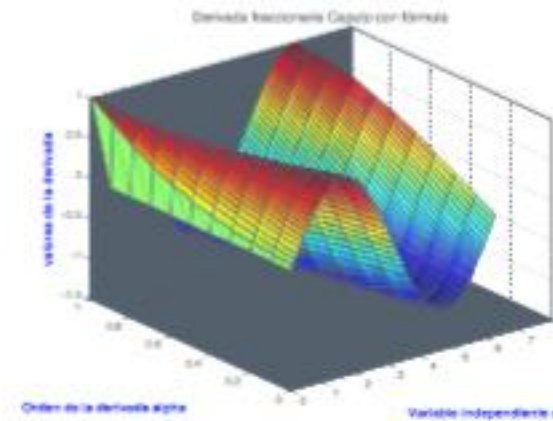


(b) orden de 1 a 2

Figura: Derivadas fraccionarias de seno y exponencial, donde el orden va de 0 a 1



(a) exponencial e^x



(b) seno $\sin x$

3.- Fractal Geometry versus Fractional Calculus.

Let $C=C_n$ be the Cantor set, the fractal dimension of C is $\nu=\ln 2/\ln 3$. Then if $f(x)$ has domain at C and $K = \lim_{n \rightarrow \infty} K_n$ are the Cantor Bars we will obtain the ν - fractional derivative of $f(x)$ in the next way:

$$K = \lim_{n \rightarrow \infty} K_n$$

$$K(x) * f(x) = \lim_{n \rightarrow \infty} K_n * f(x)$$

$$\mathcal{L}\{K(x) * f(x)\} = \mathcal{L}\{K(x)\} \mathcal{L}\{f(x)\} = \lim_{n \rightarrow \infty} \prod_0^n \left(1 + e^{-[y(1-\xi)\xi^n]} \right) \left(\frac{1 - e^{-y\xi^n}}{y\xi^n} \right) (\mathcal{L}\{f\})(y)$$

$$\frac{C(\nu)}{\Gamma(\nu)} \int_0^x (x - \tau)^{\nu-1} f(\tau) d\tau = C(\nu) D^{-\nu} f(x)$$

Análisis Fraccionario.

Carlos Fuentes, Anthony Torres.

- Durante la Pandemia 2020-2022. Aplicaciones de Análisis Fraccionario.
- Petroleo, Finanzas (Black-Schols) con funciones de Base Radial, Newton-Rhapson Fraccionario, Paneles Solares , Inversión en epocas de incertidumbre, Soluciones de la Zeta de Riemann (Números Primos), Comportamiento de datos COVID.

Producción en la pandemia 2020, 2021

- Grupo de Análisis Fraccionario:
- Carlos Fuentes, IMTA, Fernando Alcantara UAQ. Pedro Rodrigo, Jaen España, UP-A. Anthony Torres, UNAM. Carlos Torres, UACM. Benito Martines, UAEM.
- 5 tesis doctorales.
- 23 artículos de investigación. 47 citas.
- Se ofrecerá: Curso de posgrado Análisis Fraccionario, UNAM 24-1.

1 Introducción

- Método de Newton-Raphson
- Cálculo Fraccional

2 Métodos Iterativos Fraccionales

- Newton-Raphson Fraccional
- Newton Fraccional
- Pseudo-Newton Fraccional

3 Algunas Aplicaciones del Cálculo Fraccional

- Paneles Solares Híbridos
- Inversión en Épocas de Incertidumbre
- Función zeta de Riemann

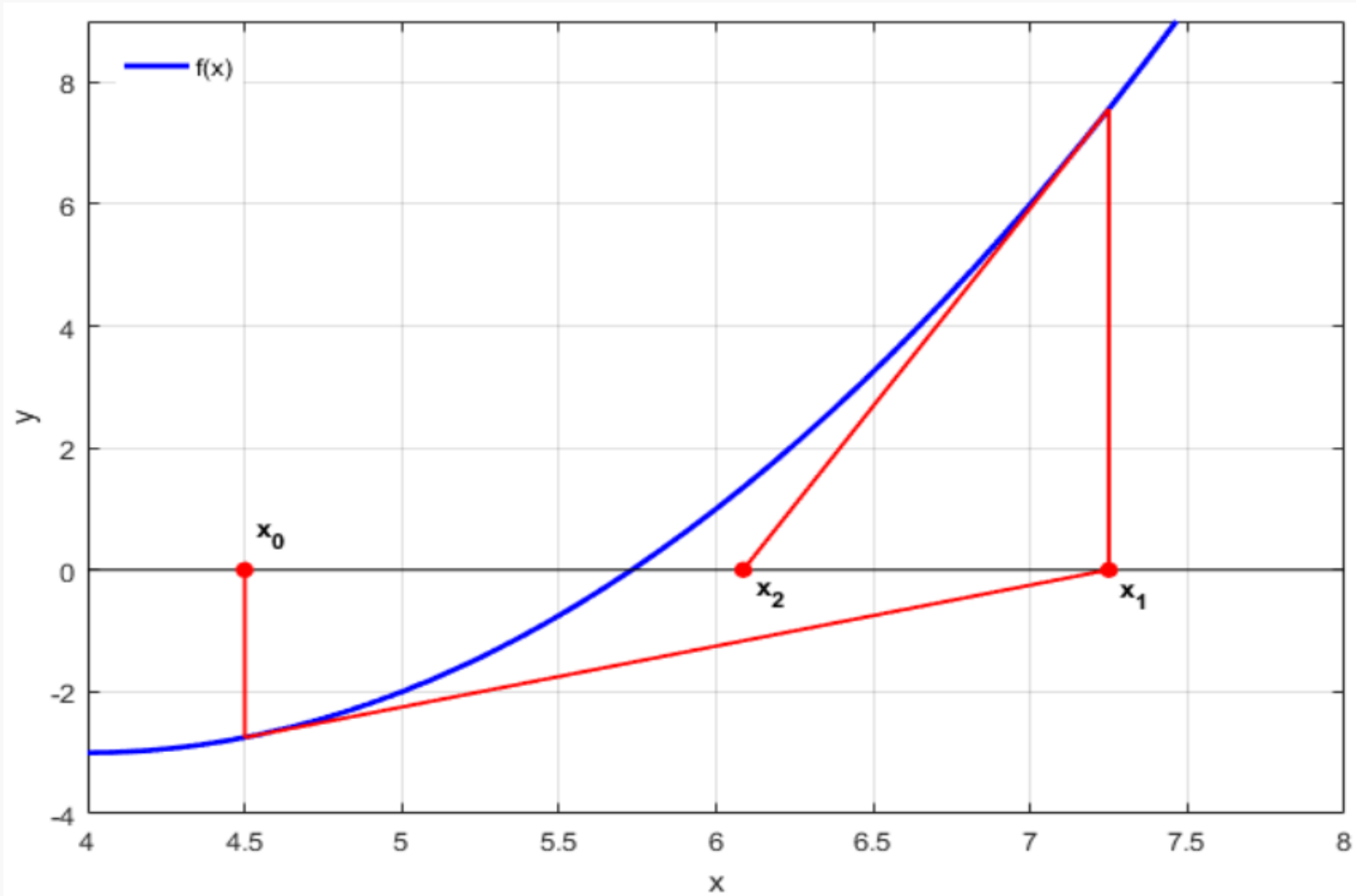
4 Bibliografía

Método de Newton-Raphson

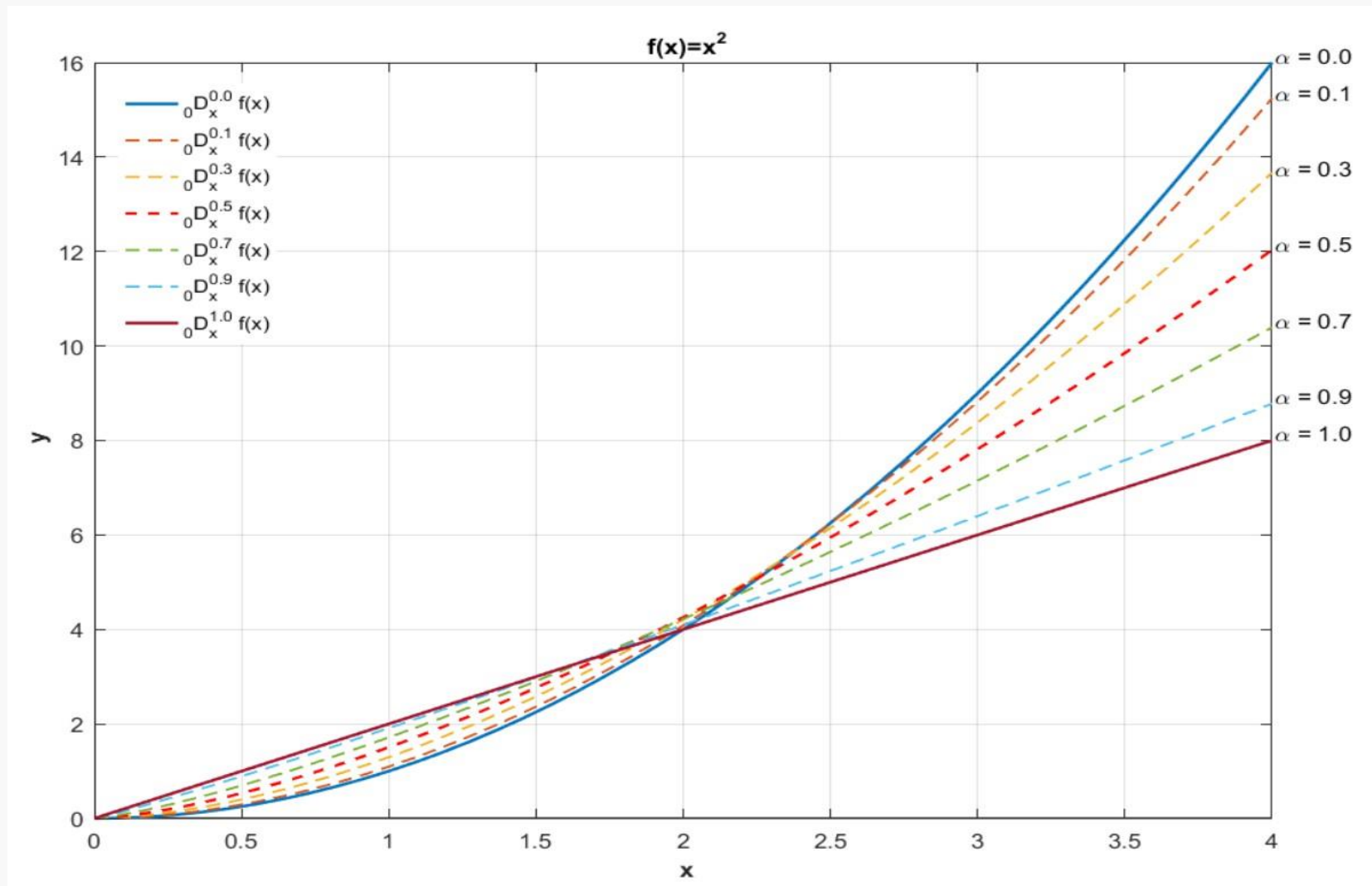
- Joseph Raphson fue el primero en publicar el método en 1690 en su libro *Analysis Aequationum Universalis*, y el de Newton se publicó unos años después de su muerte, en 1736 (por lo que Raphson lo publicó casi 50 años antes).

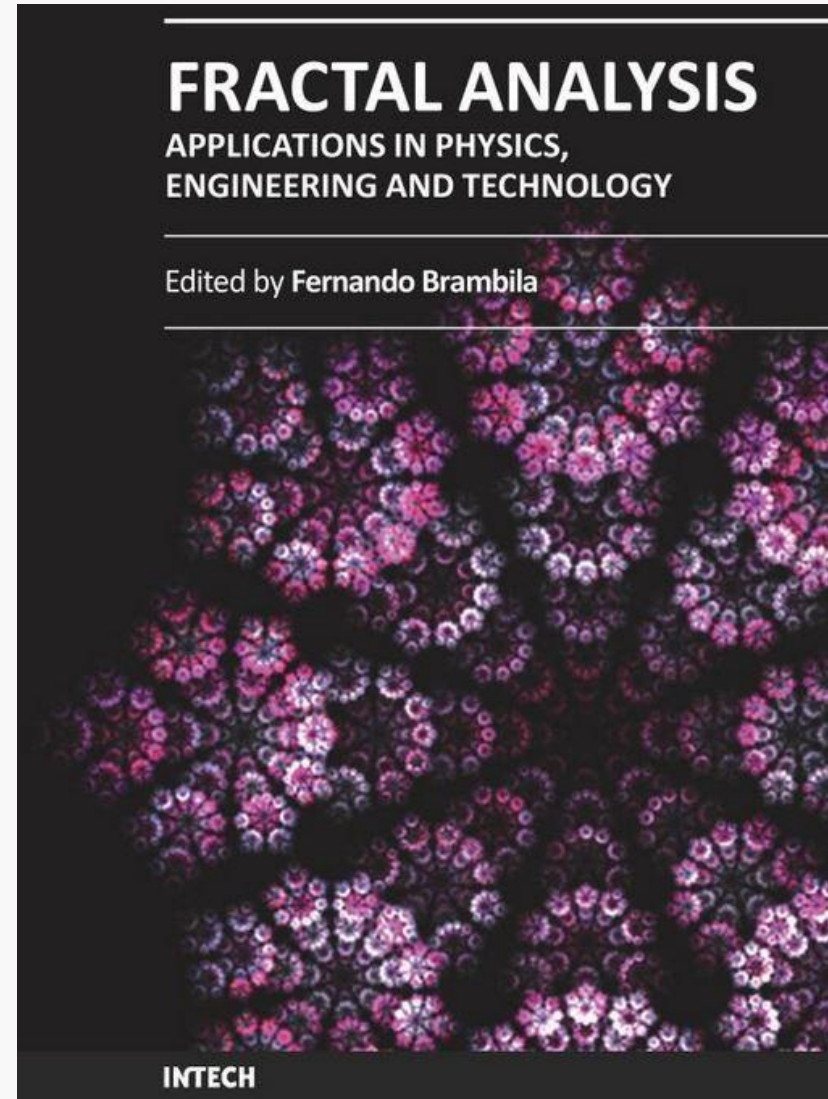


Método de Newton-Raphson



Cálculo Fraccional





Fractional Newton-Raphson Method Accelerated with Aitken's Method

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ursula@ciencias.unam.mx

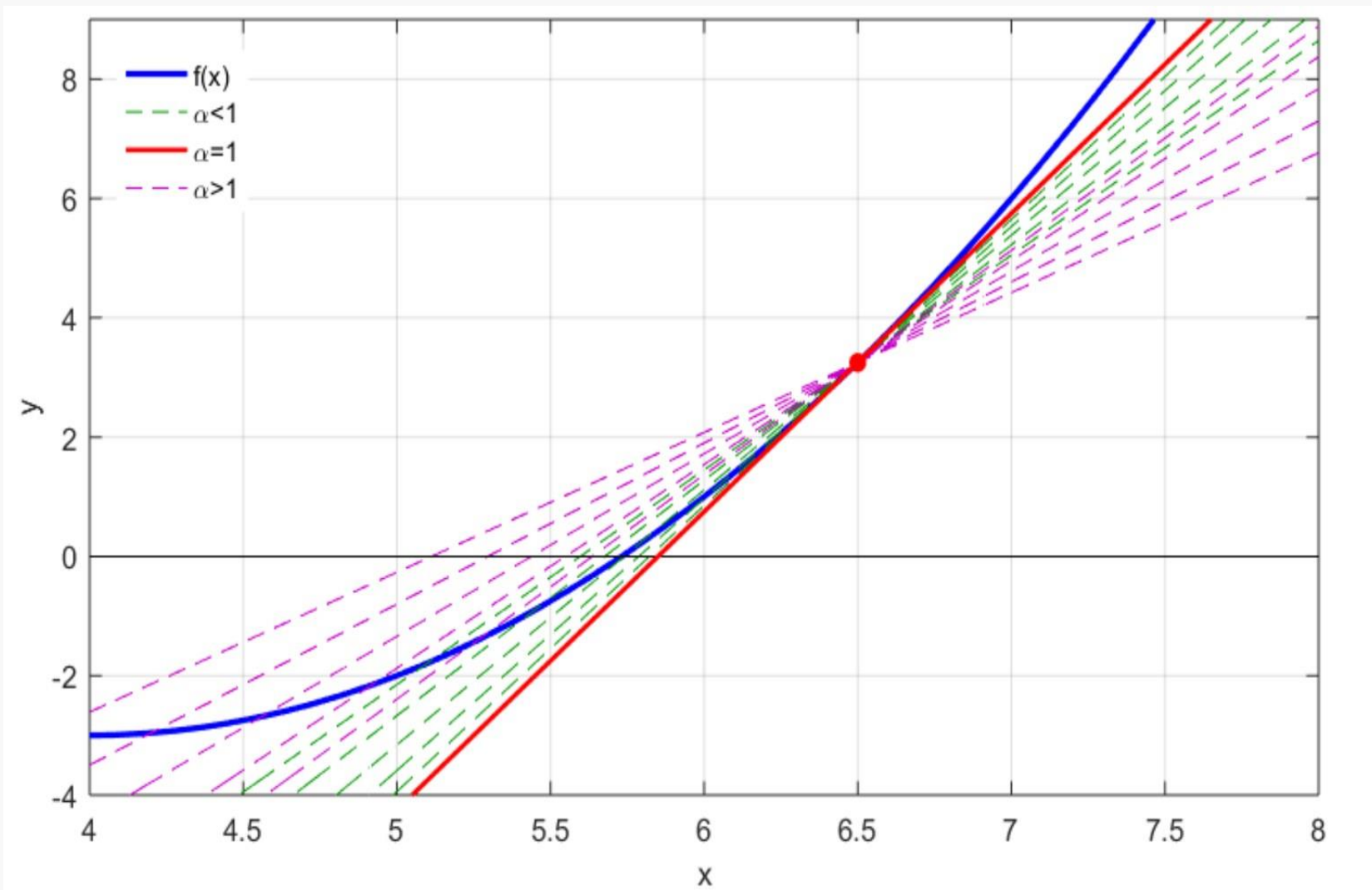
A. TORRES-HERNANDEZ

Department of Physics - UNAM
anthony.torres@ciencias.unam.mx

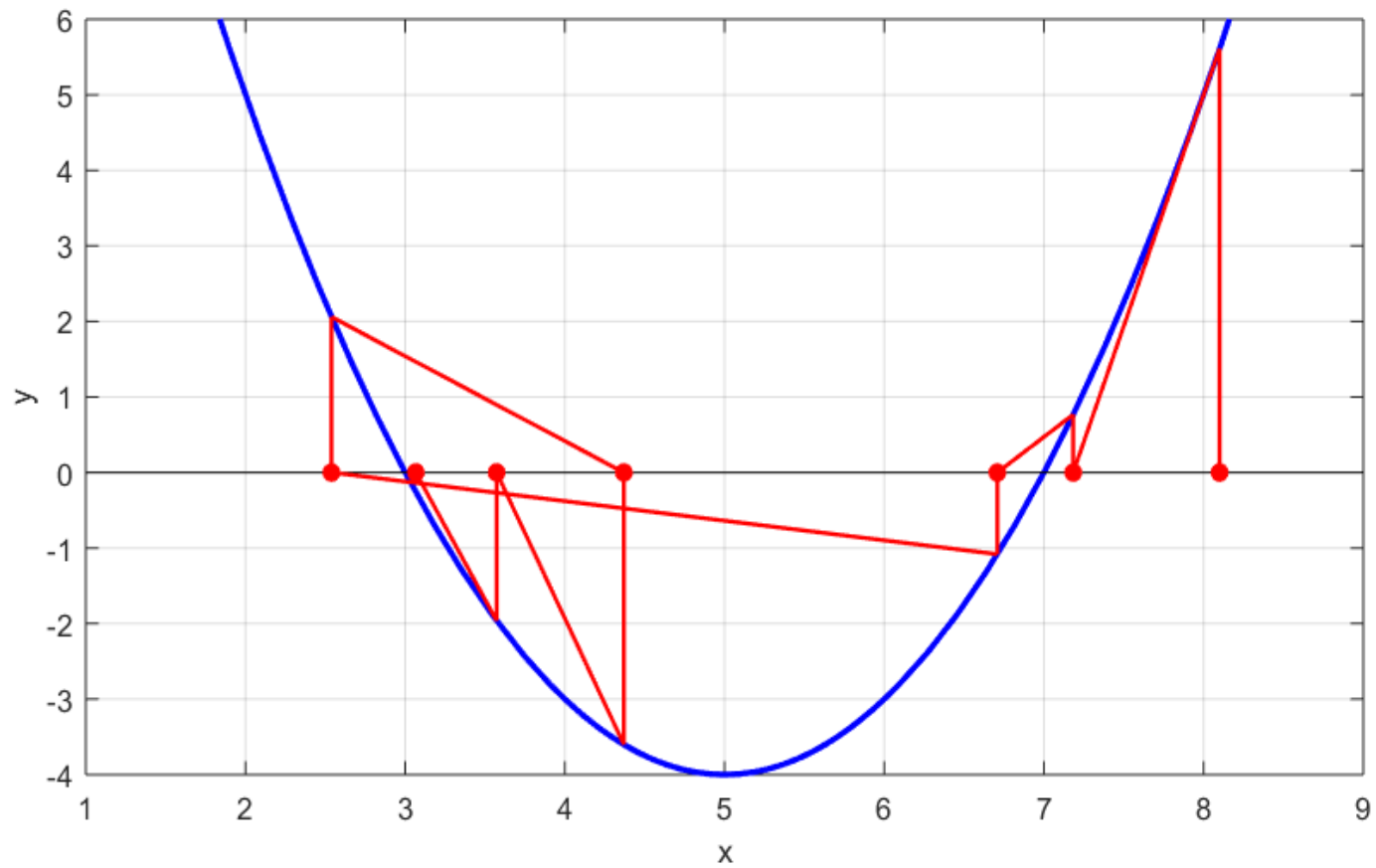
R. CABALLERO-CRUZ

Department of Physics - UNAM
rcaballero@ciencias.unam.mx

Newton Fraccional

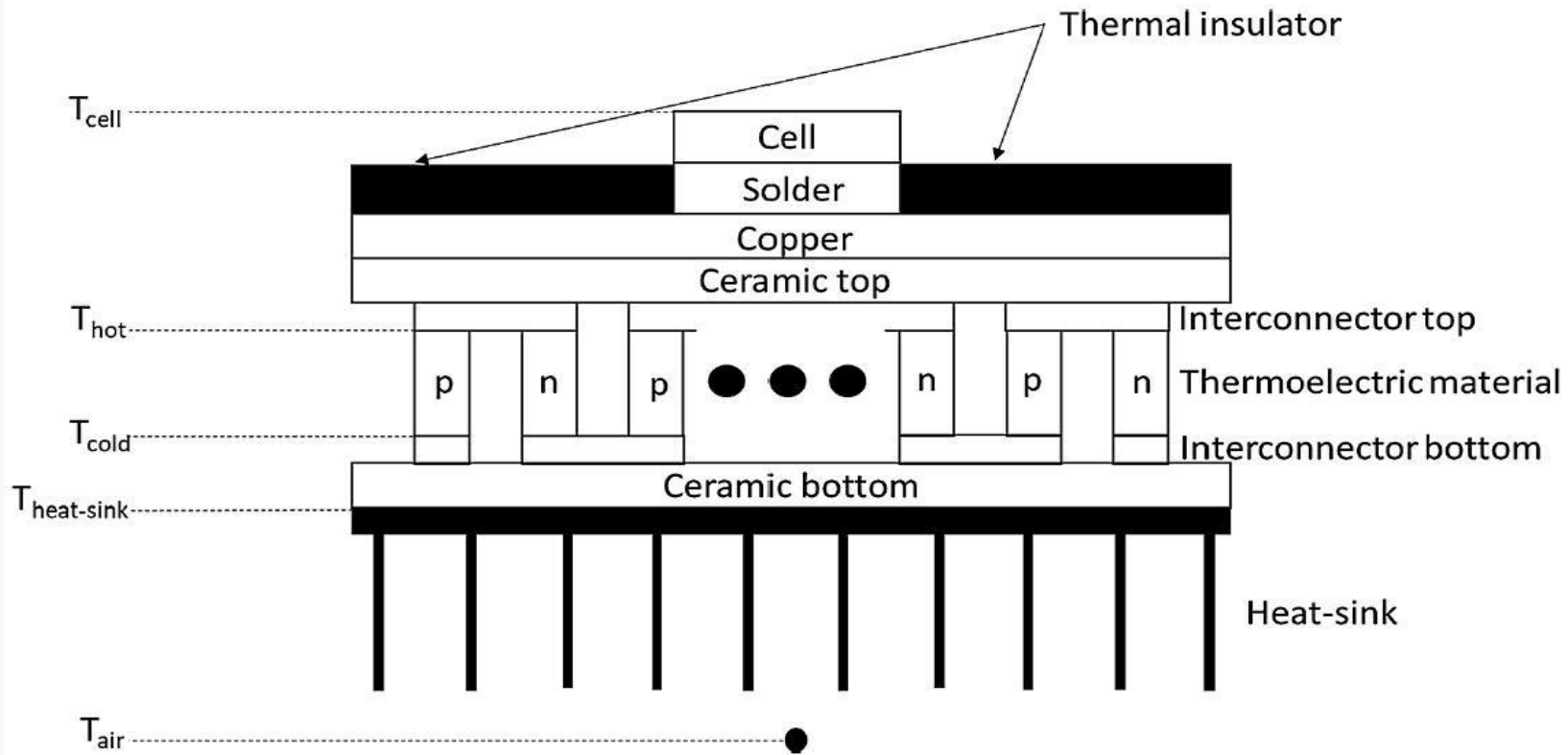


Newton Fraccional



c) $\alpha = 0.19$

Paneles Solares Híbridos



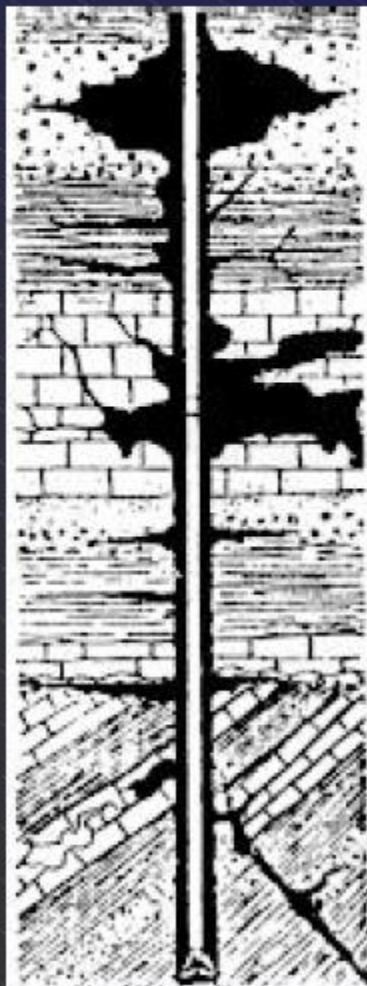
3.- Inverse problems in fractional Partial Differential Equations.

C. Fuentes.

- Triple porosity and permeability model (matrix, vugs, and fractures systems).
- Transfer Model (Fuentes, Brambila, Vauclin et al., 2001)
- Modelo físico
 - Para la realización del modelo se hace uso de:
 - La Ecuación de Continuidad (Ley de la conservación de la masa).
 - Ley de Darcy.
 - Ecuaciones de Navier-Stokes.
 - Geometría del medio.
 - Condiciones iniciales y de frontera.

Los yacimientos son sistemas heterogéneos

Las heterogeneidades se presentan en diferentes escalas



Sistema poroso en rocas carbonatadas:
matriz - vùgulo - fractura



vùgulo

matriz

fractura

ECUACIONES PARA EL MEDIO TRIÁDICO EN LA VECINDAD DE UN POZO Coordenadas Polares

Matriz:

$$\phi_m c_m \frac{\partial p_m}{\partial t} = \frac{k_m}{\mu} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p_m}{\partial r} \right) + a_{mf} (p_f - p_m) + a_{mv} (p_v - p_m)$$

Fractura:

$$\phi_f c_f \frac{\partial p_f}{\partial t} = \frac{k_f}{\mu} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p_f}{\partial r} \right) - a_{mf} (p_f - p_m) + a_{fv} (p_v - p_f)$$

Vúgulo:

$$\phi_v c_v \frac{\partial p_v}{\partial t} = \frac{k_v}{\mu} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial p_v}{\partial r} \right) - a_{mv} (p_v - p_m) - a_{fv} (p_v - p_f)$$

La porosidad y flujo compuestos:

$$\phi = \phi_m + \phi_f + \phi_v$$

$$q = q_m + q_f + q_v$$

YACIMIENTOS PETROLEROS

Ecuación General del Flujo Monofásico Saturado:

$$\phi c \frac{\partial p}{\partial t} = \nabla \cdot \left(\frac{k}{\mu} \nabla p \right) + \gamma$$

Coefficiente de Compresibilidad:

$$c = \frac{1}{\phi} \frac{\partial \phi}{\partial p}$$

ECUACIONES DE FLUJO FRACCIONARIO

Ecuaciones con derivada temporal y derivada espacial fraccionaria.

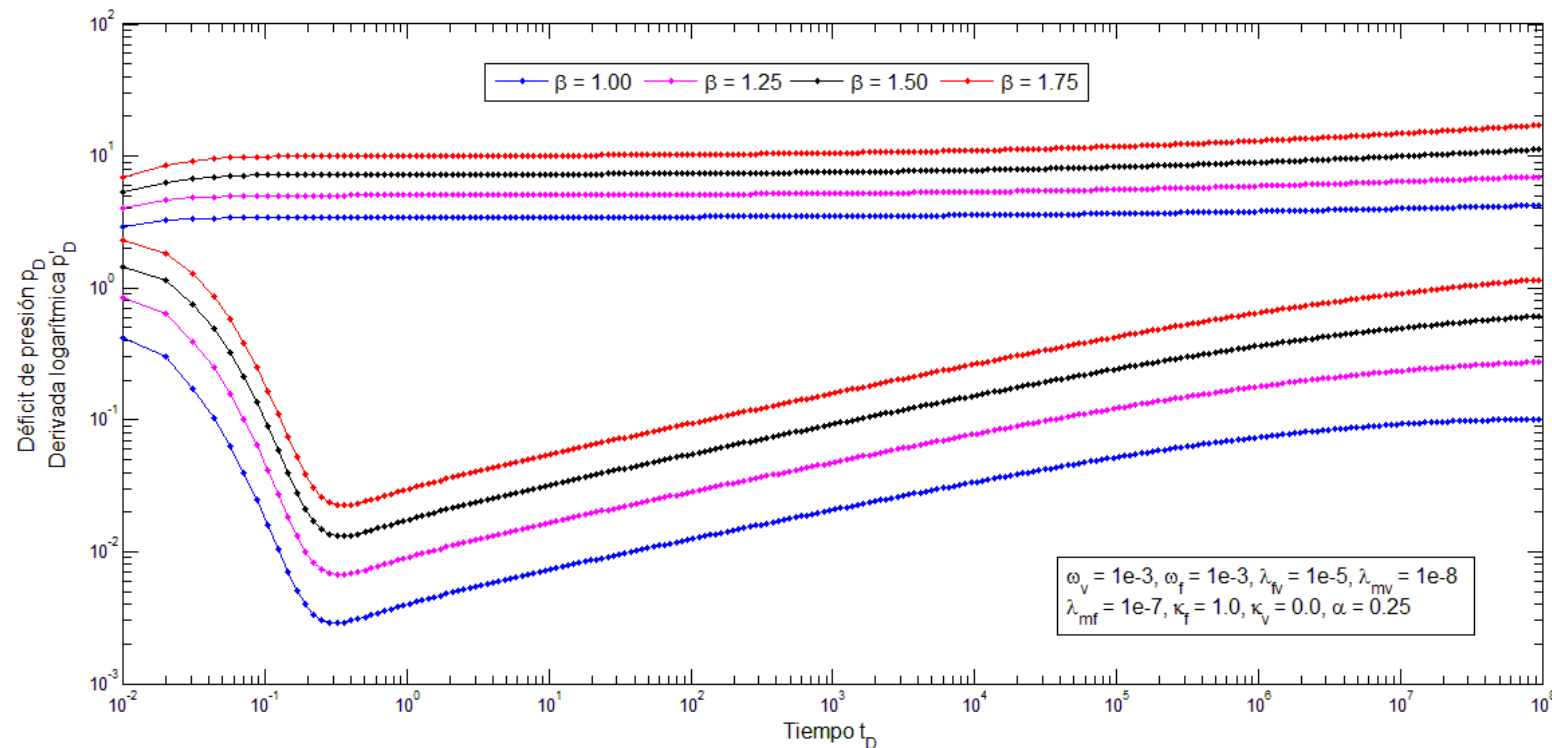
$$(1 - \omega_f - \omega_v) \frac{\partial^{\alpha_1} p_{Dm}}{\partial t_D^{\alpha_1}} = (1 - \kappa_f - \kappa_v) \frac{1}{r_D} \frac{\partial}{\partial r_D} \left(r_D \frac{\partial^{\beta_1} p_{Dm}}{\partial r_D^{\beta_1}} \right) + \lambda_{mf} (p_{Df} - p_{Dm}) + \lambda_{mv} (p_{Dv} - p_{Dm})$$

$$\omega_f \frac{\partial^{\alpha_2} p_{Df}}{\partial t_D^{\alpha_2}} = \kappa_f \frac{1}{r_D} \frac{\partial}{\partial r_D} \left(r_D \frac{\partial^{\beta_2} p_{Df}}{\partial r_D^{\beta_2}} \right) - \lambda_{mf} (p_{Df} - p_{Dm}) + \lambda_{fv} (p_{Dv} - p_{Df})$$

$$\omega_v \frac{\partial^{\alpha_3} p_{Dv}}{\partial t_D^{\alpha_3}} = \kappa_v \frac{1}{r_D} \frac{\partial}{\partial r_D} \left(r_D \frac{\partial^{\beta_3} p_{Dv}}{\partial r_D^{\beta_3}} \right) - \lambda_{mv} (p_{Dv} - p_{Dm}) - \lambda_{fv} (p_{Dv} - p_{Df})$$

DERIVADA ESPACIAL Y DERIVADA TEMPORAL FRACCIONARIA

Se presenta la solución al modelo de triple porosidad y triple permeabilidad combinando la derivada temporal fraccionaria con el gradiente fraccionario.





Dedicado a:

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GRACIAS!

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