ESTIMACIÓN DE LA CURVA DE CONDUCTIVIDAD HIDRÁULICA DEL SUELO A PARTIR DE SU CURVA DE RETENCIÓN DE AGUA

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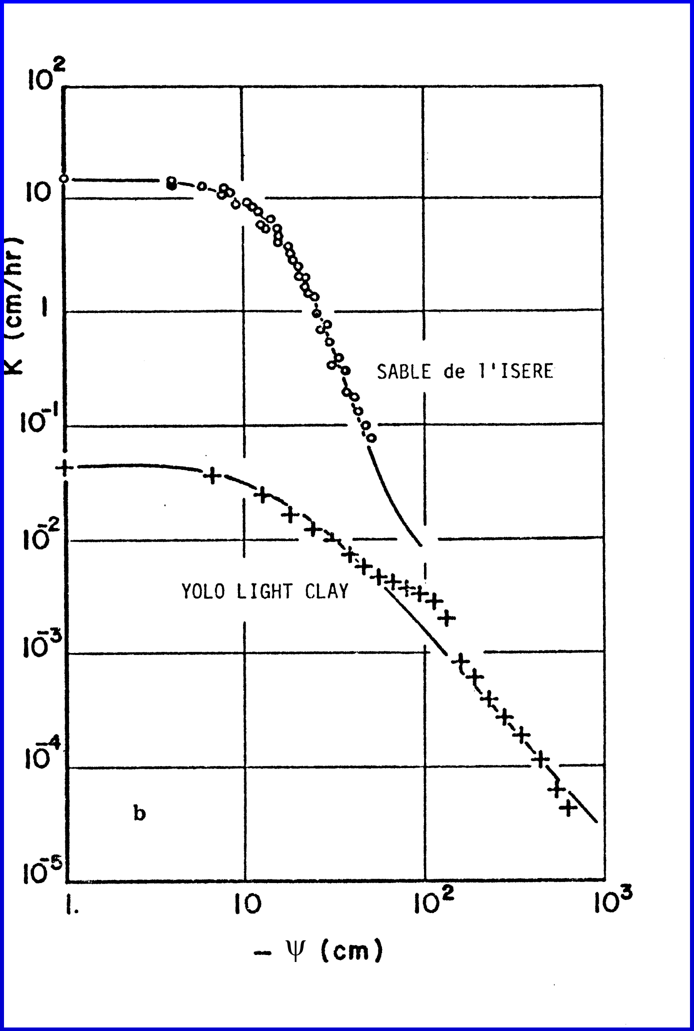
LA ECUACIÓN DE CONTINUIDAD

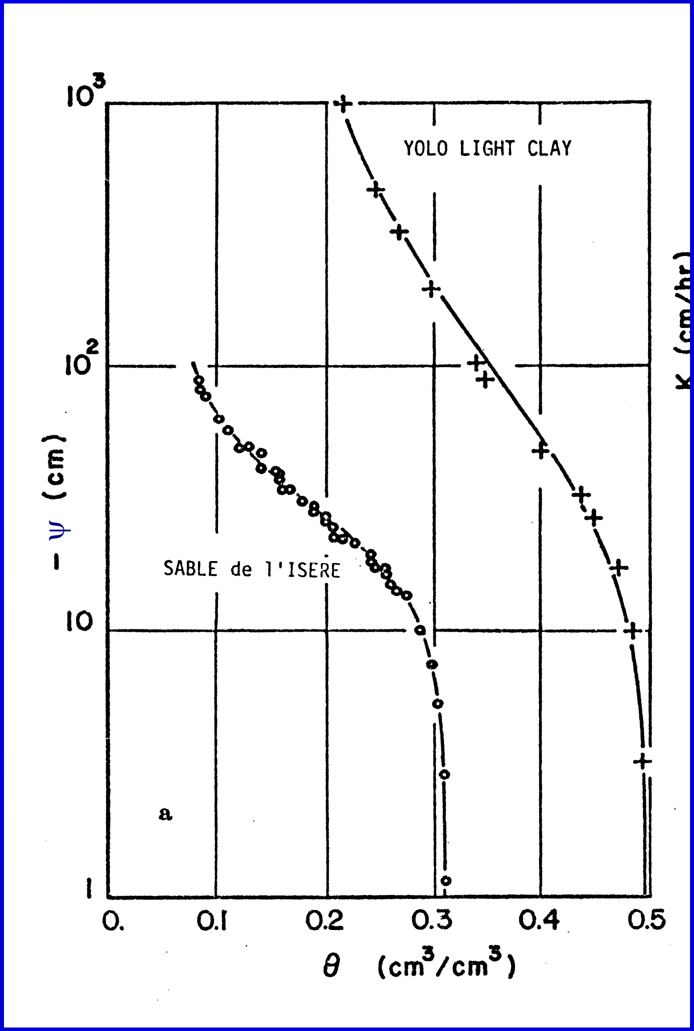
$$\frac{\partial \theta}{\partial t} + \mathrm{div}(q) + R = 0$$

LA LEY DE DARCY

$$q = -K(\psi)\nabla H \quad ; \quad H = \psi - z$$

CARACTERÍSTICAS HIDRODINÁMICAS





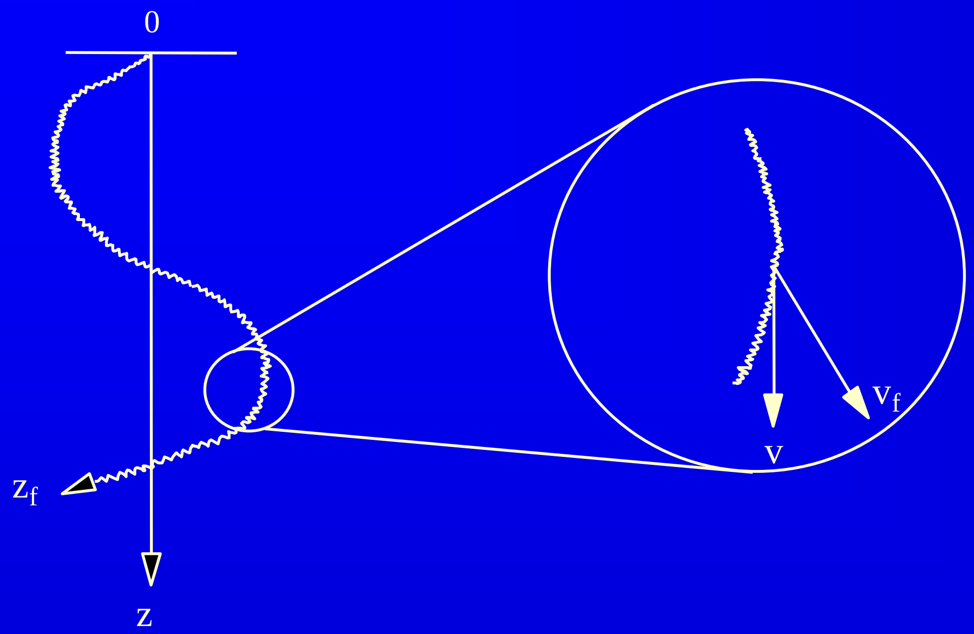
LA LEY DE POISEUILLE

$$v = -C_f \frac{\rho_w g}{\eta} R^2 \nabla H$$

LA LEY DE LAPLACE-JURIN

$$R = -\frac{2\sigma \cos(\alpha)}{\rho_w g \psi}$$

EL CONCEPTO DE TORTUOSIDAD



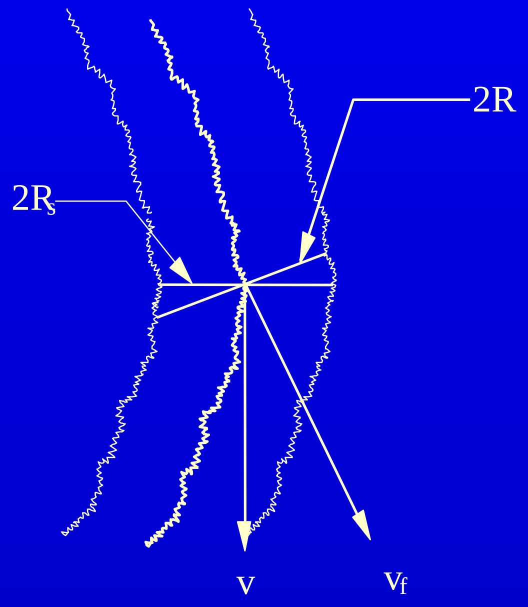
$$v = \frac{dz}{dt}$$

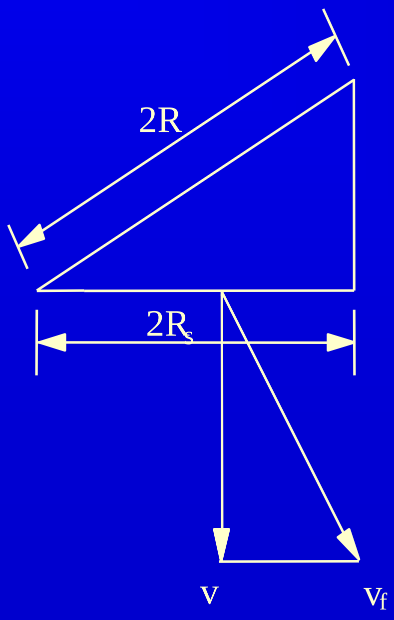
$$v_f = \frac{dz_f}{dt}$$

$$T = \frac{dz_f}{dz} = \frac{v_f}{v} \geq 1$$

LA LEY DE POISEUILLE GENERALIZADA

$$v = -C_f \frac{\rho_w g}{\eta} \left(\frac{R}{T} \right)^2 \frac{\partial H}{\partial z}$$





$$T = \frac{v_f}{v} = \frac{R}{R_s}$$

DE LA LEY DE POISEUILLE A LA LEY DE DARCY

$$q = \int_{\Omega} v\, d\omega = -C_f \frac{\rho_w g}{\eta} \frac{\partial H}{\partial z} \int_{\Omega} R_s^2 d\omega = -K \frac{\partial H}{\partial z}$$

Conductividad:

$$K = \frac{\rho_w g}{\eta} k$$

Permeabilidad:

$$k = C_f \int_{\Omega} R_s^2 d\omega$$

$$R_s = \frac{R}{T}$$

LA POROSIDAD DEL SUELO

$$\text{Porosidad Volumétrica} = \frac{\text{volumen de vacíos}}{\text{volumen total}}$$

$$\int_{\Omega_T} d\theta = \phi$$

$$\text{Porosidad Areal} = \frac{\text{área de vacíos}}{\text{área total}}$$

$$\int_{\Omega_T} d\omega = \mu$$

MODELO CONCEPTUAL DE LA CONDUCTIVIDAD

Permeabilidad:

$$k = C_f \int_{\Omega} R_s^2 \, d\omega$$

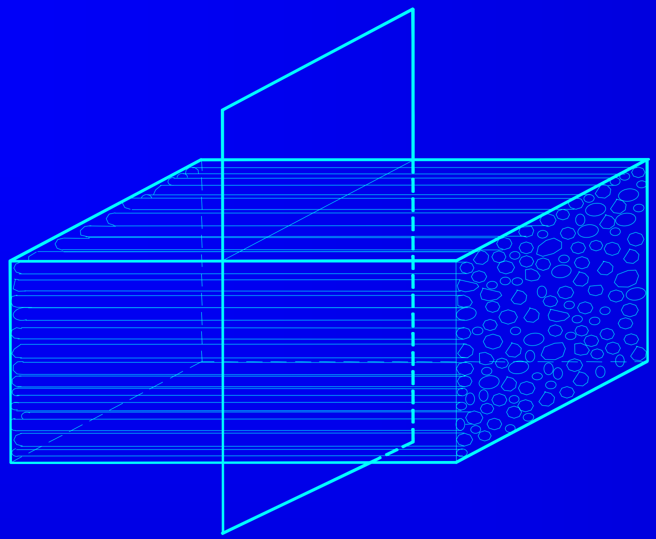
Permeabilidad Total:

$$k_s = C_f \int_{\Omega_T} R_s^2 \, d\omega \propto \mu$$

Permeabilidad Relativa:

$$k / k_s = \int_{\Omega} R_s^2 \, d\omega \Big/ \int_{\Omega_T} R_s^2 \, d\omega$$

MODELO DE PURCELL (1949)



Porosidad: $d\omega(r) = d\theta(r) = f(r)dr, \qquad \mu = \phi$

Tortuosidad: $T = T_o = \text{constante}$

Permeabilidad: $k = \dfrac{C_f}{T_o^2}\int_0^{\theta} r^2 d\vartheta$

MODELO DE BURDINE (1953)

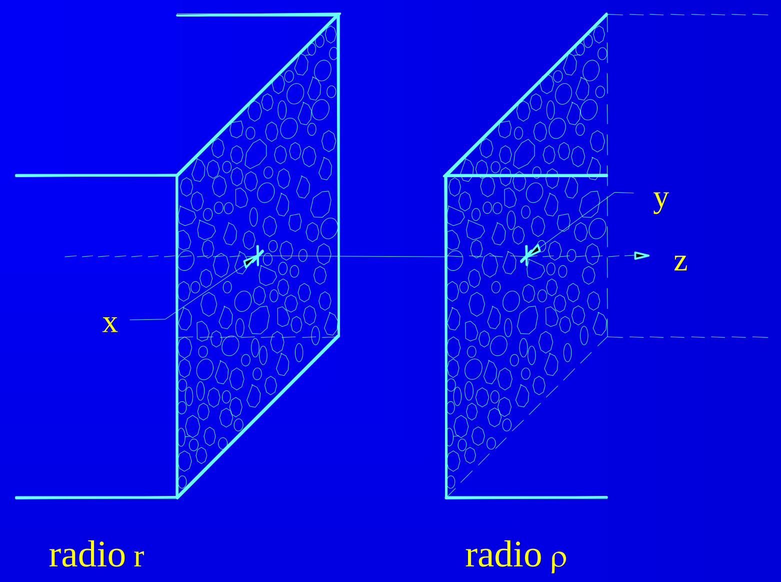
Tortuosidad (empírica):

$$T = T_o \left(\frac{\phi}{\theta} \right)$$

Permeabilidad:

$$k \propto \left(\frac{\theta}{\phi} \right)^2 \int_0^\theta r^2 d\vartheta$$

MODELO DE CHILDS Y COLLIS-GEORGE (1950)-1



Porosidad: $$d\omega(r,\rho) = d\theta(r)d\theta(\rho) = f(r)drf(\rho)d\rho.$$

$$\mu = \phi^2$$

MODELO DE CHILDS Y COLLIS-GEORGE (1950)-2

$$k = C_f \iint_{\Omega} \left(R/T\right)^2 d\theta(r) d\theta(\rho) = C_f \iint_{\Omega} \left(R/T\right)^2 f(r) dr f(\rho) d\rho.$$

Hipótesis: $R = \min(r, \rho)$

Tortuosidad: $T = T_o = \text{constante}$

Permeabilidad: $k = \dfrac{2C_f}{T_o^2} \displaystyle\int_0^{\theta} [\theta - \vartheta] r^2 d\vartheta$

MODELO DE MUALEM (1976)

Hipótesis:

$$R^2 = r\rho$$

Permeabilidad:

$$k \propto \left(\frac{\theta}{\phi}\right)^{1/2}\left[\int_0^{\theta} r\,d\vartheta\right]^2$$

MODELO DE FUENTES (1992)

Hipótesis:

$$R = \max\left(r, \rho\right)$$

Permeabilidad:

$$k \propto \left[\frac{\theta}{\phi}\right]^{p} \int_{0}^{\theta} r^{2} \vartheta \, d\vartheta$$

RESUMEN DE MODELOS CLÁSICOS GENERALIZADOS-1

Poro Pequeño:

$$R = \min\left(r, \rho\right)$$

$$\frac{K(\theta)}{K_s} = \left[\frac{\theta}{\phi}\right]^p \left[\int_0^{\theta} \frac{\theta - \vartheta}{\psi^2(\vartheta)} d\vartheta \Big/ \int_0^{\phi} \frac{\phi - \vartheta}{\psi^2(\vartheta)} d\vartheta\right]$$

Poro Geométrico:

$$R^2 = r\rho$$

$$\frac{K(\theta)}{K_s} = \left[\frac{\theta}{\phi}\right]^p \left[\int_0^{\theta} \frac{d\vartheta}{\psi(\vartheta)} \Big/ \int_0^{\phi} \frac{d\vartheta}{\psi(\vartheta)}\right]^2$$

RESUMEN DE MODELOS CLÁSICOS GENERALIZADOS-2

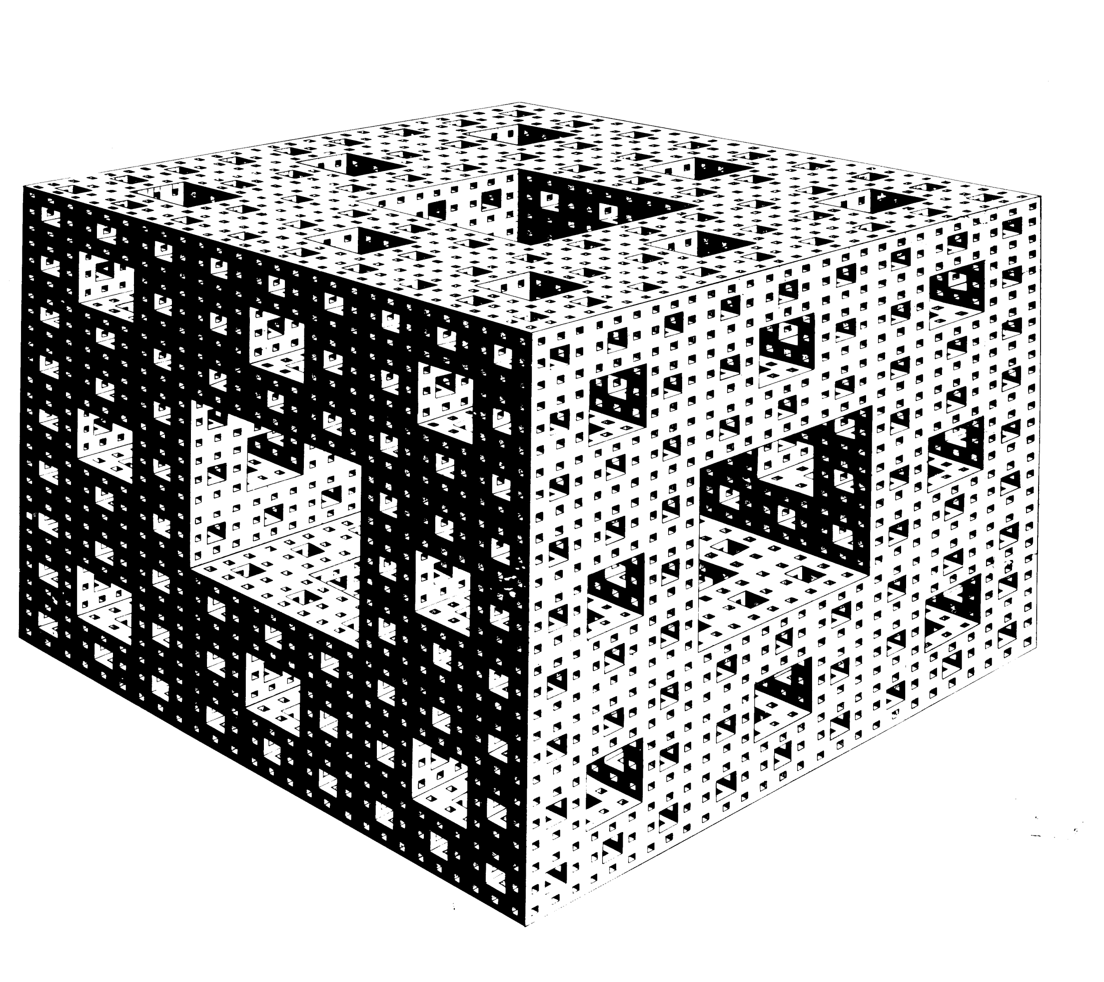
Poro Neutro: $R = r \quad o \quad R = \rho$

$$\frac{K(\theta)}{K_s} = \left[\frac{\theta}{\phi}\right]^{p+1} \left[\int_0^{\theta} \frac{d\vartheta}{\psi^2(\vartheta)} \Big/ \int_0^{\phi} \frac{d\vartheta}{\psi^2(\vartheta)} \right].$$

Poro Grande: $R = \max(r, \rho)$

$$\frac{K(\theta)}{K_s} = \left[\frac{\theta}{\phi}\right]^{p} \left[\int_0^{\theta} \frac{\vartheta}{\psi^2(\vartheta)} d\vartheta \Big/ \int_0^{\phi} \frac{\vartheta}{\psi^2(\vartheta)} d\vartheta \right].$$

CONCEPTOS DE LA GEOMETRÍA FRACTAL-7



CONCEPTOS DE LA GEOMETRÍA FRACTAL-4

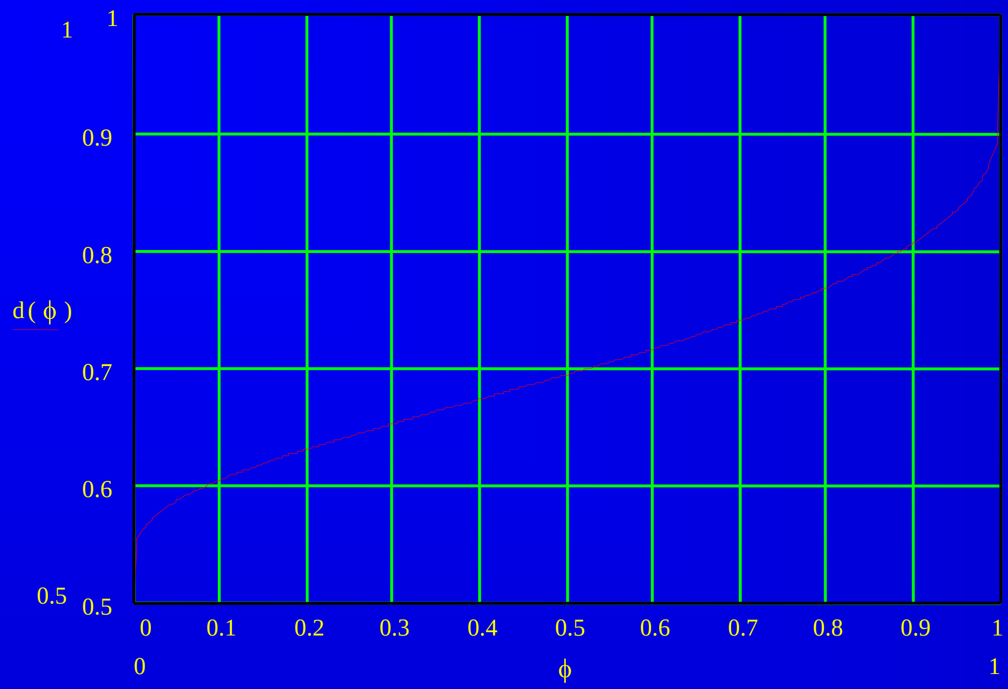
La Medida de Hausdorff-Mandelbrot

$$H^D = \lim_{r \to 0} \left[N_r r^D \right]$$

La Dimensión de Mandelbrot

$$D = \lim_{r \to 0} \left[\frac{\log(N_r)}{\log(H/r)} \right].$$

POROSIDAD VS DIMENSIÓN-2

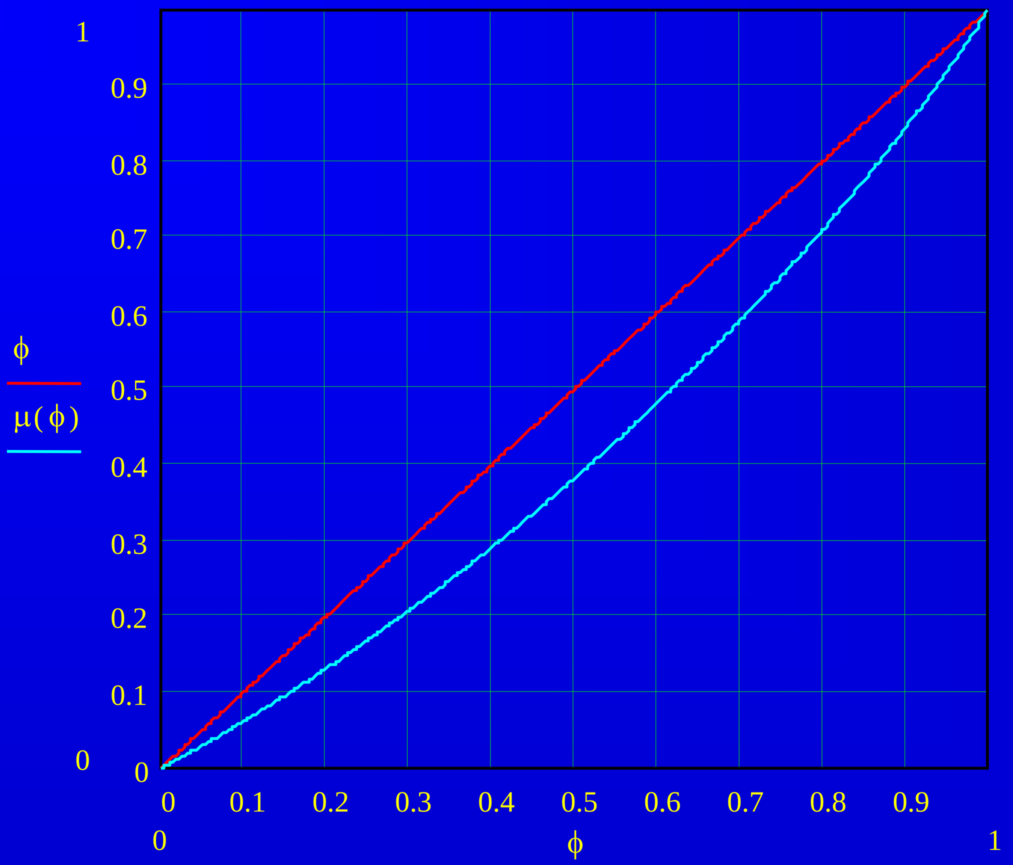


$$(1-\phi)^{s} + \phi^{2s} = 1$$

$$1/2 < s < 1$$

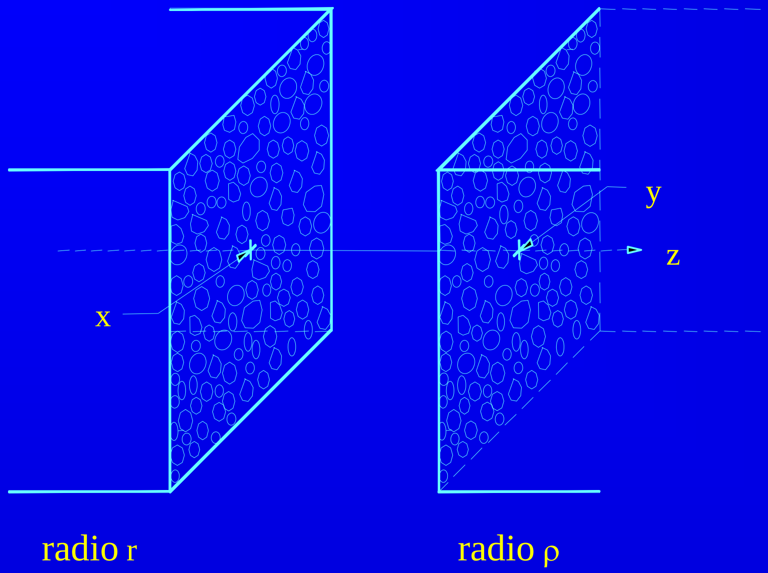
$$E/2 < D < E$$

POROSIDAD VS DIMENSIÓN-3



$$\mu = \phi^{2s}$$

MODELO FRACTAL DEL ÁREA DE FLUJO



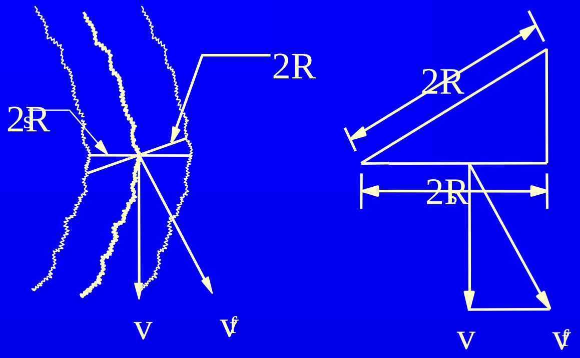
$$0 \leq \theta \leq \phi$$

$$0 \leq \omega \leq \mu$$

Porosidad Areal Total: $\mu = \phi^{2s} = \phi^{s}\phi^{s}$

Porosidad Areal Parcial: $d\omega(r,\rho) = d\theta^{s}(r)\,d\theta^{s}(\rho)$

MODELACIÓN FRACTAL DE LA TORTUOSIDAD



$$\theta(R) = \phi S(R/R_o)$$

$$\omega(R_s) = \mu S(R_s/R_{so})$$

$$S(r) = r^{\lambda}$$

$$T = \frac{v_f}{v} = \frac{R}{R_s}$$

$$\omega = \theta^{2s}$$

$$\mu = \phi^{2s}$$

$$\frac{R_s}{R_{so}} = \left(\frac{R}{R_o}\right)^{2s}$$

$$T = T_o\left(\frac{R_o}{R}\right)^{2s-1}$$

MODELO FRACTAL DE LA CONDUCTIVIDAD

$$k = C_f \int_{\Omega} \left(R/T\right)^2 d\omega$$

$$d\omega(r,\rho) = d\theta^s\left(r\right)d\theta^s\left(\rho\right)$$

$$T = T_o\left(\frac{R_o}{R}\right)^{2s-1}$$

$$\left(1-\phi\right)^s + \phi^{2s} = 1$$

MODELOS FRACTALES DE LA CONDUCTIVIDAD-1

Poro Pequeño:

$$R = \min(r, \rho)$$

$$\frac{K(\theta)}{K_s} = \int_0^{\theta} \frac{\theta^s - \vartheta^s}{[\psi(\vartheta)]^{4s}} \vartheta^{s-1} d\vartheta \Bigg/ \int_0^{\phi} \frac{\phi^s - \vartheta^s}{[\psi(\vartheta)]^{4s}} \vartheta^{s-1} d\vartheta$$

Poro Geométrico:

$$R^2 = r\rho$$

$$\frac{K(\theta)}{K_s} = \left[\int_0^{\theta} \frac{\vartheta^{s-1} d\vartheta}{[\psi(\vartheta)]^{2s}} \Bigg/ \int_0^{\phi} \frac{\vartheta^{s-1} d\vartheta}{[\psi(\vartheta)]^{2s}} \right]^2 .$$

MODELOS FRACTALES DE LA CONDUCTIVIDAD-2

Poro Neutro: $R = r \quad o \quad R = \rho$

$$\frac{K(\theta)}{K_s} = \left[\frac{\theta}{\phi}\right]^s \left[\int_0^{\theta} \frac{\vartheta^{s-1} d\vartheta}{[\psi(\vartheta)]^{4s}} \Big/ \int_0^{\phi} \frac{\vartheta^{s-1} d\vartheta}{[\psi(\vartheta)]^{4s}}\right]$$

Poro Grande: $R = \max(r, \rho)$

$$\frac{K(\theta)}{K_s} = \int_0^{\theta} \frac{\vartheta^{2s-1}}{[\psi(\vartheta)]^{4s}} d\vartheta \Big/ \int_0^{\phi} \frac{\vartheta^{2s-1}}{[\psi(\vartheta)]^{4s}} d\vartheta$$

MODELO CLÁSICOS FRACTALES

$$k = C_f \int_{\Omega} \left(R/T \right)^2 d\omega$$

$$d\omega(r, \rho; R) = \theta^{2s-2}(R)\, d\theta(r)\, d\theta(\rho)$$

$$T(R) = T_o \left[\frac{\phi}{\theta(R)} \right]^{(2s-1)/3(1-s)}$$

$$(1-\phi)^s + \phi^{2s} = 1$$

MODELOS CLÁSICOS FRACTALES-1

Poro Pequeño:

$$R = \min\left(r, \rho\right)$$

$$\frac{K(\theta)}{K_s} = \left[\frac{\theta}{\phi}\right]^p \left[\int_0^\theta \frac{\theta - \vartheta}{\psi^2(\vartheta)} d\vartheta \Bigg/ \int_0^\phi \frac{\phi - \vartheta}{\psi^2(\vartheta)} d\vartheta\right]$$

Poro Geométrico:

$$R^2 = r\rho$$

$$\frac{K(\theta)}{K_s} = \left[\frac{\theta}{\phi}\right]^p \left[\int_0^\theta \frac{d\vartheta}{\psi(\vartheta)} \Bigg/ \int_0^\phi \frac{d\vartheta}{\psi(\vartheta)}\right]^2$$

MODELOS CLÁSICOS FRACTALES-2

Poro Neutro: $R = r \quad o \quad R = \rho$

$$\frac{K(\theta)}{K_s} = \left[\frac{\theta}{\phi}\right]^{p+1} \left[\int_0^{\theta} \frac{d\vartheta}{\psi^2(\vartheta)} \Big/ \int_0^{\phi} \frac{d\vartheta}{\psi^2(\vartheta)}\right].$$

Poro Grande: $R = \max(r, \rho)$

$$\frac{K(\theta)}{K_s} = \left[\frac{\theta}{\phi}\right]^{p} \left[\int_0^{\theta} \frac{\vartheta}{\psi^2(\vartheta)} d\vartheta \Big/ \int_0^{\phi} \frac{\vartheta}{\psi^2(\vartheta)} d\vartheta\right].$$

EL PARÁMETRO P

$$p = p_1 + p_2$$

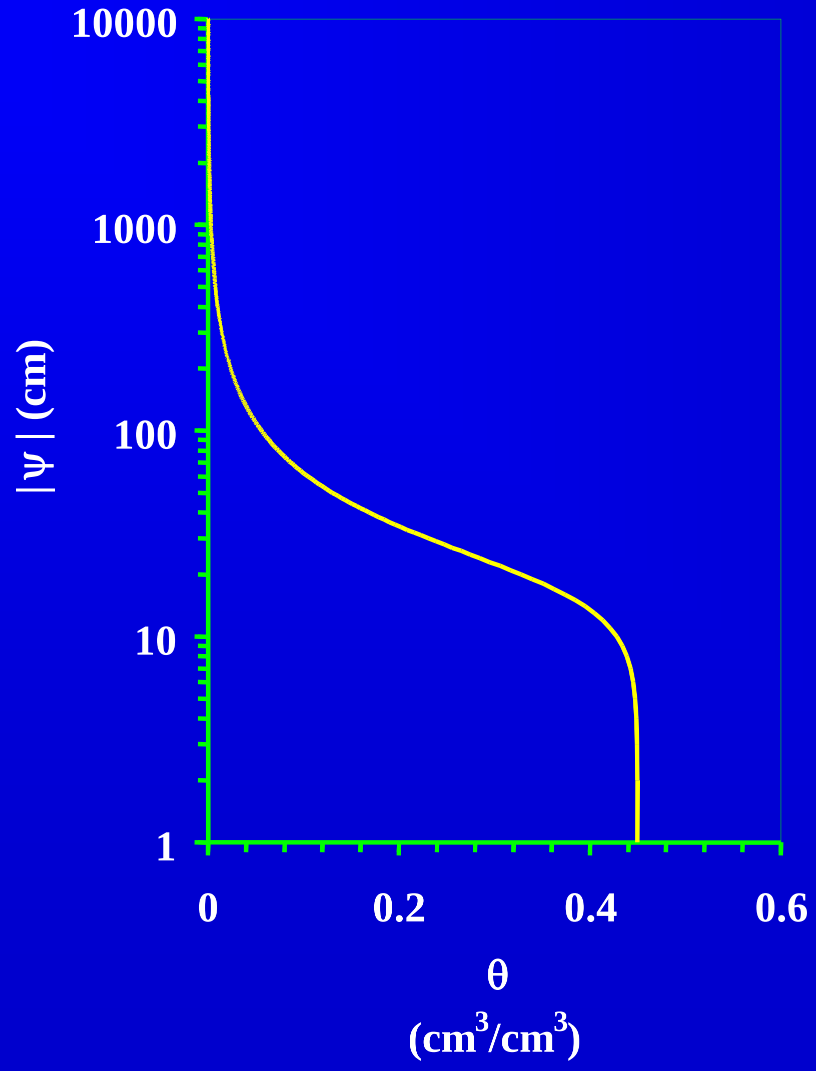
Correlación: $p_1 = 2s - 2$ **Tortuosidad:** $p_2 = \dfrac{2(2s-1)}{3(1-s)}$

ϕ	$s = D/3$	p_1	p_2	$p = p_1 + p_2$
0	1/2	-1	0	-1
0.3671	2/3	-2/3	2/3	0
½	0.6942	-0.6115	0.8470	0.2355
0.6180	0.7202	-0.5596	1.0494	0.4898
1	1	0	∞	∞

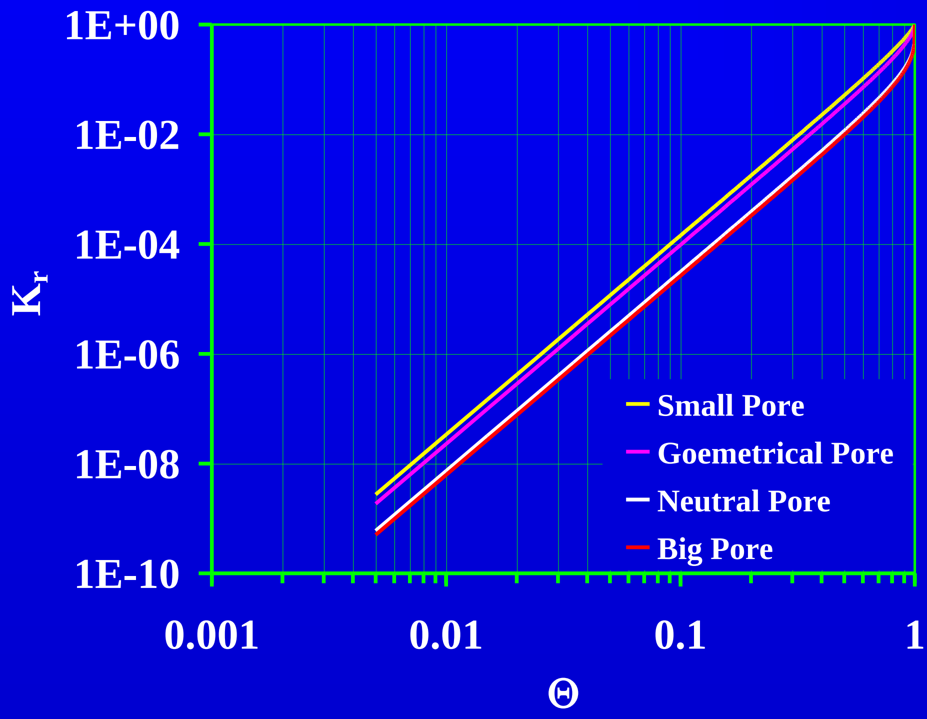
Van Genuchten model of the retention curve

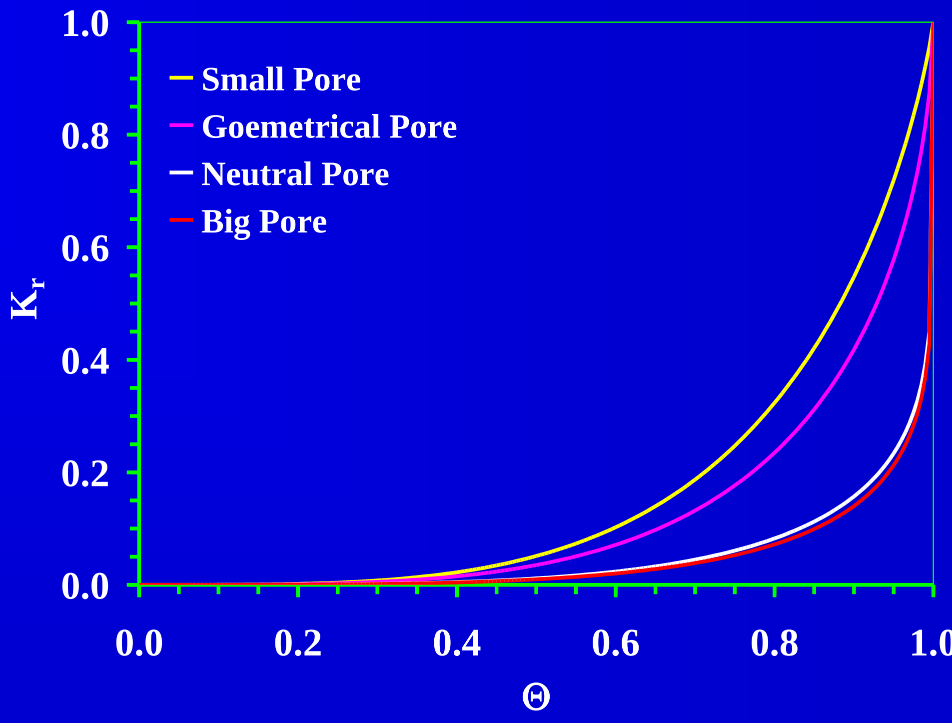
$$\theta(\psi) = \frac{\theta_s - \theta_r}{\left[1 + \left(\frac{\psi}{\psi_d}\right)^n\right]^m} + \theta_r$$

THE RETENTION CURVE



THE FOUR MODELS OF THE HYDRAULIC CONDUCTIVITY





CONCLUSIONS

1. We have deduced the fractal unification of the capilarity models of the hydraulic conductivity of Darcy Law from the Poiseuille Law.

2. The unification of the classical models of the hydraulic conductivity depends on the hypothesis on the equality between pore volume and paralell body volume.

3. The experimental validation will depend on the information quality of the hydraulic conductivity associated to the retention curve.